

06-09-00

410 Rec'd PCT/PTO 0 8 JUN 2000

FORM PTO-1390 (Modified)  
(REV 11-98)

U.S. DEPARTMENT OF COMMERCE PATENT AND TRADEMARK OFFICE

ATTORNEY'S DOCKET NUMBER

## TRANSMITTAL LETTER TO THE UNITED STATES

GCLL-115

DESIGNATED/ELECTED OFFICE (DO/EO/US)

U.S. APPLICATION NO. (IF KNOWN, SEE 37 CFR

CONCERNING A FILING UNDER 35 U.S.C. 371

09/581095

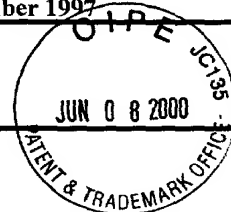
INTERNATIONAL APPLICATION NO.  
PCT/US98/26384INTERNATIONAL FILING DATE  
11 December 1998PRIORITY DATE CLAIMED  
12 December 1997

TITLE OF INVENTION

METHOD AND SYSTEM FOR NESTING OBJECTS

APPLICANT(S) FOR DO/EO/US

Isaac SADOVIK



Applicant herewith submits to the United States Designated/Elected Office (DO/EO/US) the following items and other information:

1. ☒ This is a **FIRST** submission of items concerning a filing under 35 U.S.C. 371.
2. ☐ This is a **SECOND** or **SUBSEQUENT** submission of items concerning a filing under 35 U.S.C. 371.
3. ☐ This is an express request to begin national examination procedures (35 U.S.C. 371(f)) at any time rather than delay examination until the expiration of the applicable time limit set in 35 U.S.C. 371(b) and PCT Articles 22 and 39(1).
4. ☒ A proper Demand for International Preliminary Examination was made by the 19th month from the earliest claimed priority date.
5. ☐ A copy of the International Application as filed (35 U.S.C. 371 (c) (2))
  - a. ☐ is transmitted herewith (required only if not transmitted by the International Bureau).
  - b. ☐ has been transmitted by the International Bureau.
  - c. ☒ is not required, as the application was filed in the United States Receiving Office (RO/US).
6. ☐ A translation of the International Application into English (35 U.S.C. 371(c)(2)).
7. ☒ A copy of the International Search Report (PCT/ISA/210).
8. ☐ Amendments to the claims of the International Application under PCT Article 19 (35 U.S.C. 371 (c)(3))
  - a. ☐ are transmitted herewith (required only if not transmitted by the International Bureau).
  - b. ☐ have been transmitted by the International Bureau.
  - c. ☐ have not been made; however, the time limit for making such amendments has NOT expired.
  - d. ☐ have not been made and will not be made.
9. ☐ A translation of the amendments to the claims under PCT Article 19 (35 U.S.C. 371(c)(3)).
10. ☐ An oath or declaration of the inventor(s) (35 U.S.C. 371 (c)(4)).
11. ☒ A copy of the International Preliminary Examination Report (PCT/IPEA/409).
12. ☐ A translation of the annexes to the International Preliminary Examination Report under PCT Article 36 (35 U.S.C. 371 (c)(5)).

## Items 13 to 20 below concern document(s) or information included:

13. ☐ An Information Disclosure Statement under 37 CFR 1.97 and 1.98.
14. ☐ An assignment document for recording. A separate cover sheet in compliance with 37 CFR 3.28 and 3.31 is included.
15. ☐ A **FIRST** preliminary amendment.
16. ☐ A **SECOND** or **SUBSEQUENT** preliminary amendment.
17. ☐ A substitute specification.
18. ☒ A change of power of attorney and/or address letter.
19. ☒ Certificate of Mailing by Express Mail
20. ☐ Other items or information:

U.S. APPLICATION NO. (IF KNOWN) SEE 37 CFR <b>09/581095</b>	INTERNATIONAL APPLICATION NO. <b>PCT/US98/26384</b>	ATTORNEY'S DOCKET NUMBER <b>GCLL-115</b>
--	--	---

21. The following fees are submitted.:

**BASIC NATIONAL FEE ( 37 CFR 1.492 (a) (1) - (5)) :**

- |  |                 |
|--|-----------------|
| <input type="checkbox"/> Neither international preliminary examination fee (37 CFR 1.482) nor international search fee (37 CFR 1.445(a)(2)) paid to USPTO and International Search Report not prepared by the EPO or JPO ..... | <b>\$970.00</b> |
| <input type="checkbox"/> International preliminary examination fee (37 CFR 1.482) not paid to USPTO but International Search Report prepared by the EPO or JPO .....   | <b>\$840.00</b> |
| <input type="checkbox"/> International preliminary examination fee (37 CFR 1.482) not paid to USPTO but international search fee (37 CFR 1.445(a)(2)) paid to USPTO .....  | <b>\$690.00</b> |
| <input type="checkbox"/> International preliminary examination fee paid to USPTO (37 CFR 1.482) but all claims did not satisfy provisions of PCT Article 33(1)-(4) .....   | <b>\$670.00</b> |
| <input checked="" type="checkbox"/> International preliminary examination fee paid to USPTO (37 CFR 1.482) and all claims satisfied provisions of PCT Article 33(1)-(4) .....  | <b>\$96.00</b>  |

**ENTER APPROPRIATE BASIC FEE AMOUNT =****\$96.00**

Surcharge of **\$130.00** for furnishing the oath or declaration later than ☐ 20 ☒ 30 months from the earliest claimed priority date (37 CFR 1.492 (e)).

**\$130.00**

CLAIMS	NUMBER FILED	NUMBER EXTRA	RATE		
Total claims	6 - 20 =	0	x \$18.00		<b>\$0.00</b>
Independent claims	3 - 3 =	0	x \$78.00		<b>\$0.00</b>
Multiple Dependent Claims (check if applicable).				<input type="checkbox"/>	<b>\$0.00</b>

**TOTAL OF ABOVE CALCULATIONS =****\$226.00**

Reduction of 1/2 for filing by small entity, if applicable. Verified Small Entity Statement must also be filed (Note 37 CFR 1.9, 1.27, 1.28) (check if applicable). ☒

**\$113.00****SUBTOTAL =****\$113.00**

Processing fee of **\$130.00** for furnishing the English translation later than ☐ 20 ☐ 30 months from the earliest claimed priority date (37 CFR 1.492 (f)).

**\$0.00****TOTAL NATIONAL FEE =****\$113.00**

Fee for recording the enclosed assignment (37 CFR 1.21(h)). The assignment must be accompanied by an appropriate cover sheet (37 CFR 3.28, 3.31) (check if applicable). ☐

**\$0.00****TOTAL FEES ENCLOSED =****\$113.00**

Amount to be: refunded	\$
charged	\$

☒ A check in the amount of **\$113.00** to cover the above fees is enclosed.

☐ Please charge my Deposit Account No. \_\_\_\_\_ in the amount of \_\_\_\_\_ to cover the above fees.  
A duplicate copy of this sheet is enclosed.

☒ The Commissioner is hereby authorized to charge any fees which may be required, or credit any overpayment to Deposit Account No. **50-1133** A duplicate copy of this sheet is enclosed.

**NOTE: Where an appropriate time limit under 37 CFR 1.494 or 1.495 has not been met, a petition to revive (37 CFR 1.137(a) or (b)) must be filed and granted to restore the application to pending status.**

SEND ALL CORRESPONDENCE TO:

**Mark G. Lappin**  
**McDermott, Will & Emery**  
 28 State Street  
 Boston, MA 02109  
 Tel (617) 535-4043  
 Fax (617) 535-3800

SIGNATURE

**Mark G. Lappin**

NAME

**26,618**

REGISTRATION NUMBER

**June 8, 2000**

DATE

**IN THE UNITED STATES PATENT AND TRADEMARK OFFICE**

**APPLICANT(S):** Isaac Sadovnik  
**SERIAL NO.** Not Assigned  
**FILED:** Herewith  
**ENTITLED:** METHOD AND SYSTEMS FOR NESTING OBJECTS

**VERIFIED STATEMENT (DECLARATION) CLAIMING SMALL ENTITY STATUS  
(37 CFR 1.9(f) and 1.27(b))—SMALL BUSINESS CONCERN**

I hereby declare that I am

- ☐ the owner of the small business concern identified below:  
☒ an official of the small business concern empowered to act on behalf of the concern identified below:

**NAME OF CONCERN** NesTech, Inc.  
**ADDRESS OF CONCERN** 801 Main Street, Concord, MA 01742

I hereby declare that the above-identified small business concern qualifies as a small business as defined in 13 C.F.R. ' 121.3-18, and reproduced in 37 C.F.R. ' 1.9(d), for purposes of paying reduced fees under Sections 41(a) and (b) of Title 35, United States Code, in that the number of employees of the concern, including those of its affiliates, does not exceed 500 persons. For purposes of this statement, (1) the number of employees of the business concern is the average over the previous fiscal year of the concern of the persons employed on a full-time, part-time or temporary basis during each of the pay periods of the fiscal year, and (2) concerns are affiliates of each other when either directly, or indirectly, one concern controls or has the power to control the other, or a third party controls or has the power to control both.

I hereby declare that rights under contract or law have been conveyed to and remain with the small business concern identified above with regard to the invention described in:

- ☒ the specification filed herewith with title listed above.  
☐ the application identified above.  
☐ the patent identified above.

If the rights held by the above identified small business concern are not exclusive, each individual, concern or organization having rights in the invention must file separate statements as to their status as small entities, and no rights to the invention are held by any person, other than the inventor, who would not qualify as an independent inventor under 37 CFR 1.9(c) if that person made the invention, or by any concern which would not qualify as a small business concern under 37 CFR 1.9 (d), or a nonprofit organization under 37 CFR 1.9 (e).

Each person, concern or organization having any rights in the invention is listed below:

- ☒ no such person, concern or organization exists.  
☐ each such person, concern or organization is listed below.

FULL NAME \_\_\_\_\_

ADDRESS \_\_\_\_\_

☐ Individual ☐ Small Business Concern ☐ Nonprofit Organization

Separate statements are required from each named person, concern or organization having rights to the invention averring to their status as small entities. (37 CFR 1.27)

I acknowledge the duty to file, in this application or patent, notification of any change in status resulting in loss of entitlement to small entity status prior to paying, or at the time of paying, the earliest of the issue fee or any maintenance fee due after the date on which status as a small entity is no longer appropriate (37 C.F.R. 1.28(b)).

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code, and that such willful false statements may jeopardize the validity of the application, any patent issue thereon, or any patent to which this verified statement is directed.

NAME OF PERSON SIGNING Gerard F. Uva

TITLE OF PERSON OTHER THAN OWNER Vice President & Chief Financial Officer

ADDRESS OF PERSON SIGNING 801 Main Street  
Concord, MA 01742

SIGNATURE



DATE June 8, 2000

416 Rec'd PCT/PTO 0 8 JUN 2000

10

**I. METHOD AND SYSTEMS FOR NESTING TWO DIMENSIONAL  
OBJECTS IN A TARGET PLANE**

**FIELD OF THE INVENTION**

5           The present invention is in the field of manufacturing processing and more particularly relates to methods and systems for nesting of two dimensional objects. Nesting is the process of placing objects together with minimal space between them.

**BACKGROUND OF THE INVENTION**

10           There are many design and manufacturing processes in which substantially two dimensional parts-to-be-assembled are initially laid out on and then cut from planar or sheet stock. For example, leather shoes, clothing, pocketbooks, and automobile and furniture upholstery are often formed from a plurality of panels, or parts, cut from a cowhide. After cutting of the panels for such an article,  
15           conventional seam joining techniques, such as sewing, are used to assemble the panels to form the finished article. Clothing, upholstery and many other articles of other materials are similarly designed and manufactured in a similar manner. Such other articles include body parts for automobiles, for example, hoods or fenders, which may be stamped from metal sheets, and then subsequently shaped to form the  
20           final part configuration. Numerous other articles are similarly manufactured from planar sheets as well.

          In the manufacture of all of these parts, it is important to minimize the waste which remains after the parts are cut from the sheet stock. While in some cases, where the waste can be re-used, such as in the case of sheet metal, this factor is of  
25           lesser importance, there are many cases where the minimization of waste is of critical importance to the commercial viability of a product. The latter cases particularly include shoe manufacture, where the cost of materials, for example cowhides or synthetic materials, is very high.

          In view of such cost considerations, manufacturing processes generally  
30           include attempts to minimize waste. However, the conventional attempts at such minimization have been relatively ineffective. In the leather shoe industry, for example, parts are often manually die cut from cowhides, where the die is manually

placed on the hide by skilled (or relatively unskilled) workers, typically resulting in maximum yields in the 50-60% range for the typical irregularly shaped parts. In recent years, computer controlled layout of parts on a hide have been achieved, resulting in typical yields as high as the 60-70% range.

5 In U.S. Patent No. 4,210,041, the present inventors disclosed a method for laying out, or nesting, a plurality of identical irregular non-polygonal rounded pieces on a parallel edge-bounded sheet material with relatively little waste. In accordance with the method of that patent, the salient points of the part boundary are connected by straight lines to box the part boundary in a plane polygonal figure. Then, pairs of  
 10 such plane polygonal figures are pairwise positioned to within a plane closed parallelogram. Finally, a plurality of such parallelogram are arranged side-by-side to define a multiple repeat pattern of objects equal in width to the planar sheet from which the parts are to be cut. A nest constructed in accordance with U.S. Patent 4,210,041 does offer improvement in nesting over prior art techniques; however, the  
 15 resulting nests are still less than optimal.

It is an object of the present invention to provide an improved method and system for defining a nest of two dimensional objects.

It is another object to provide an improved method and apparatus for cutting a plurality of two dimensional parts from a sheet with high yield.

## 20 SUMMARY OF THE INVENTION

The invention is a method and system for defining placement of a plurality of two-dimensional objects on a target planar surface with minimal void space between them. The invention automatically packs the maximum number of irregular parts  
 25 possible (preferably, of up to two different shapes) with minimum wasted surface area. The invention uses a heuristical approach, so that intelligent decisions are made in order to carry only a small percentage of intermediate solutions to completion. In typical cases, this enables the computation time used in the nesting process to be reduced from hours to minutes, allowing for a more interactive nesting design  
 30 environment.

The invention uses a combination of topology analysis and numerical iteration to determine an efficient layout in short computation times. The process

commences by defining a first sub-nest in a central part of the target, grouping parts to each other, and then "grows" or extends outward toward the boundaries of the target. The resultant layouts are generally parallelogramic and thus angled in nature. The "angled" layout is contrary to conventional rectangular (90 degree) nesting practice. A heuristic approach is used with an interactive process to curtail long computation times and to produce nested layouts much faster.

The nesting method and system of the invention maximizes material utilization from the "center outwards" by minimizing the voids between nested parts. In contrast, conventional nesting practice first defines a target area and then fits parts starting from one edge and "flowing" parts across the target area until the opposite edge is reached.

The heuristics used to curtail long computation times, are intermediate measures of how good a nest is. As the process proceeds from one stage to the next, measures of nesting effectiveness are calculated and only a small percentage of all the possibilities are allowed to continue to higher stages. This "nest culling" heuristical process is effective in substantially reducing computation times. By allowing only the best intermediate nests to proceed to the "tiling" stages, layout times are typically reduced with no practical loss in nesting efficiency.

## **BRIEF DESCRIPTION OF THE DRAWINGS**

The foregoing and other object of this invention, the various features thereof, as well as the invention itself, may be more fully understood from the following description, when read together with the accompanying drawings in which:

FIG. 1 shows a block diagram of a system in accordance with the invention;

FIG. 2 shows an exemplary part to be nested;

FIG. 3 shows the total nest extent for a nest of the part of FIG. 2;

FIG. 4 shows the partial nest extent for a nest of the part of FIG. 2;

FIGS. 5A and 5B shows two nest approaches for a nest of the part of FIG. 2;

FIGS. 6A-6E show the nesting procedure for the exemplary part of FIG. 2;

FIG. 7 shows a set up and actual tile for a primary tile for nests of the exemplary part of FIG. 2;

FIG. 8 illustrates the secondary tiling step of the invention; and



FIGS. 9 and 10 show exemplary nesting layouts determined using the method and system off the invention.

# **DESCRIPTION OF THE PREFERRED EMBODIMENT**

5           A system 10 embodying the invention is shown in Fig. 1. The system 10 includes a part input device 12 a programmed digital computer 14 with an associated display 16 and an output layout buffer printer 18. The part input device 12 may be a conventional computer running a computer aided design program, such as AutoCad. The computer may be an IBM type Pentium-based or an Apple PowerPC-based  
 10       personal computer (with a conventional monitor for display 16) programmed to perform the process of the invention. The layout buffer 18 may include a conventional storage medium for storing the layouts determined by the programmed digital computer 12.

          The following description sets forth how two dimensional (2D) parts are  
 15       input and defined and how their areas are computed. Data structures are then defined that hold the nesting and tiling information as well as the measure for their incremental effectiveness (permitting use of the heuristics) along the process. Finally, the following sequential steps in the procedure: (1) Nesting, (2) Primary Tiling, (3) Secondary Tiling, and (4) Layout.

20           The computer 12 uses a program that compresses 2D irregular figures (from device 12) onto a target stock area, with minimum area wasted between the figures. The program combines geometric analysis of the parts to be laid out and numerical iteration for exhaustiveness. As a main feature of the process, the pats are nested into parallelogramic orientations and not rectangular orientations (the traditional method).  
 25       This flexibility in orientation allow a larger compression of the parts and a lesser waste of material stock. In the preferred embodiment this computer program nests only two different geometrical shapes, and then "tiles" those nested pairs. The program also performs "nest culling" to avoid unnecessary computation. Major tasks performed in the invention are:

30           Task 1:       Define and implement input of the  
                           geometry of parts (components) and the  
                           target.

- Task 2: Provide an intersection algorithm/process.
- Task 3: Add multiple numerical iteration loops for the various nesting/tiling steps.
- 5 Task 4: Implement measurement of nesting success as void areas and nest extent areas.

**Task 1:**

10 The component parts (to be nested) and the target (upon which the parts are nested) are defined in two dimensional space. In this illustrated embodiment, a part can be "drawn" in the two dimensional computer screen of monitor 16 and the computer 14 then stores this information as a list of x-y coordinates for the vertices of the boundary of a piecewise linear model of the part. In other words, the parts are  
15 defined as multisided polygons. These polygons can typically have 10's or 100's of sides depending on the complexity of the figure (curvature of its sides) and the required accuracy of the nesting solution. As a result, parts are stored in files which consist of a 2 column table of x,y coordinate pairs for all the vertices of the part.

20 **Task 2:**

The intersection procedure checks for collisions between parts which may be considered to translate in two dimensional space. This is done in two dimensions by computing the smallest distance between all the vertices of a part number 1 ( $x_1, y_1$ ) and a part number 2 ( $x_2, y_2$ ). This is  $((x_2^2 - x_1^2) + (y_2^2 - y_1^2))^{1/2}$ . In addition, this check for  
25 collisions is done between every "peripheral node" of component 1 with every "peripheral node" of part 2. Once that smallest distance is obtained, it is subtracted from every node on one of the parts into what constitutes the translation of this part toward the other one until the two parts basically "touch" each other in the two dimensional space.

30 The intersection procedure constitutes a translation in a particular direction which is not controlled by the user, but is determined by the way the parts are placed in the "working area". In order to control the directionality of the

"approach/collision" process, the intersection of part 1 vertices with part 2 sides and part 2 vertices with part 1 sides is determined. The computation of vertex-to-side distances is somewhat more involved and is described below in connection with the nesting section of the process. In addition "minimum distance part-to-part  
5 approaches" are done in the X (horizontal) direction. Directionality is achieved by rotating the parts, nests or tiles a specified angle.

### Task 3:

The nesting and tiling steps have "primary" and "secondary" moves. These  
10 moves are made so as to bring two parts or two nests close to each other in order to package them as tightly as possible. The primary move is made in the X direction. The parts (or nests) are placed side by side and then one of them is offset in the Y direction. The amount of this offset (from a maximum of the part's height to the negative of the part's height) constitutes the first iteration loop. In other words, the  
15 primary move is made for every Y offset. Once the primary move is made, the secondary move is performed in the Y direction. Then a rotation of the figures is performed for the appropriate angle (90 degrees in this case) so as to bring the approaching direction in the X direction. Then, the secondary move is performed again in the X direction. This primary/secondary move iteration loop is nested within  
20 a part side-to-side combination loop. This side-side- combination loop cycles through all the possible side-side combinations of the parts nested. In addition, after every X direction move (for both primary and secondary approaches), a "tangential" move is made in the direction of the intersecting vertex/side. Again the system is rotated by the appropriate angle and another X direction approach is performed.  
25 (This slides the vertex along the intersecting side for further compactness). Finally, within the primary/secondary and tangential moves, another loop is performed which cycles through all possible vertex-to-side combinations while looking for the minimum approach intersection process set forth under Task 3 above.

### Task 4:

30 The necessary geometrical computations for the size of the nest are "Total Event" are determined in Task 4. Obtaining the area of an assembly of components

is a straight forward calculation. The extent of the void areas between the components is determined. Typically, this is a relatively complex and computer intensive calculation. However, instead of computing nest's void areas, the preferred form of the invention calculates the "Partial Extent" of a nest. As set forth below, in the nesting section of the process description, this partial extent includes a measure of the area intersected between parts when forming a nest. This area saved is the inverse or complement of the void area proposed as a heuristic. This "intersection" saved area is much easier to calculate, and has similar bearing in terms of the effectiveness of a nest as the proposed wasted void area.

The success of the heuristic-based process for nest culling is evaluated, along with total extent and partial extent. These heuristics limit the number of nests passed to the primary tiling stage, and are also applied to the primary tiling passage to secondary tiling stage. This further limits the search tree and has dramatically improved the performance of the expensive (computer time-wise) tiling stage. A system of weighting factors is used for the two main heuristic variables, so as to ascertain the relative importance of one versus the other.

### COMPUTER PROGRAM DESCRIPTION

This section provides a brief description of the program used in computer 14. The hardware platform is an Apple Power Macintosh computer with a 17" color monitor and laser printer. The CAD software used for this embodiment consists of two packages - Autodesk's AutoCAD and Graphsoft's MiniCAD. This embodiment was developed using the following combination of software tools: 1 - Macintosh Programmer's Workshop (MPW) for the development and coding environment (including editing and debugging); 2 - FORTRAN Compiler for the compilation of the program; and 3 - MiniCAD for the part input and output layout visualization. The CAD package allows ease in description on the parts, and interactive visualization of the layouts. Alternatively, AutoCAD could be used

The inner regions of the nests have great importance in the invention (as opposed to the peripheral or edge regions near the target's boundaries.) The computation is segmented into four sequential steps: Nesting, Primary Tiling, Secondary Tiling, and Layout. The process is initiated with the nesting step, where

two irregular objects or parts are taken and brought together until they "touch" each other on their periphery. This grouping of two parts is called a "nest". The parts are brought towards each other as close together as possible for form as compact as possible a nest. Depending on the complexity of the parts being nested, the computer  
5 program actually performs from many hundreds to many thousand different nests and ranks them according to their compactness (using heuristic variables previously described).

Once the nests are formed and ranked, a predetermined number of these nests (the ones that ranked highest) are passed to the primary tiling step. In this step two  
10 nests are brought together in the X direction to form a primary tile or "ptile". A ptile is made of two nests or four parts. The same rational is used to form a ptile as to form a nest. In other words, a grouping of two nests is created that forms the most compact unit. Typically, a few hundred ptiles are generated per each nest passed from the nesting step. These ptiles are consequently ranked using heuristic variables  
15 for compactness.

A small percentage of the ptiles formed (a predetermined number) are then passed onto the secondary tiling step. In this case, two ptiles are brought together in the Y direction to form a secondary tile or "stile". A stile is made up of two stiles or four nests or eight parts. Again, the same rational as before for nest formation is  
20 used, and in the process, over a thousand different stiles are generated for each of the ptiles passed onto this step from the primary tiling step. Finally, a simple layout routine takes the data generated for the best secondary tile and computes the ideal parallelogramic area needed to contain a single nest. A computation of the utilization ratio follows by calculating the actual part area and dividing that by the needed  
25 parallelogramic area. Then the layout routine creates a file with the CAD commands that generate the best layout for the parts nested.

A step by step description of the computer program will now be described, starting with a description of the parts and ending with a description of the computation steps: Nesting, Tiling and Layout.

30

### Part Description

In order to describe and input parts in two dimensions, they are defined as

polygons in the xy plane. Each polygon can be made of (typically) 100 to 1000 vertices described by x,y coordinate pairs. This method for part input enables description of parts in a simple way, which include curvatures, for the which small polygonal sides are used (vertices are very close together where the curvatures need to be described accurately).

As an example of this methodology, Fig. 2 shows an exemplary part, composed of ten vertices in the xy plane and labeled with coordinates  $(x_1, x_2)$  through  $(x_{10}, y_{10})$ . This simple figure is thus fully described with a set of 10 x,y coordinates pairs for a total of twenty numbers. The part description for the part of Fig. 2 is stored in a small text file with all the x,y coordinate pairs. Thus, design and input of parts is very simple using this mode.

### Part Area Calculation

Part areas are calculated as three different areas: (1) the actual part area, (2) the total area extent for nests and tiles, and (3) the partial area extent for nests and tiles. The computation of the part area is straight forward. If  $y_{\min}$  is the smallest y coordinate, then the part area A is given by:

$$A = \sum_{i=0}^{N_v-1} \frac{1}{2}(X_{i+1} - X_i)(Y_{i+1} + Y_i - 2Y_{\min})$$

where  $N_v$  is the number of vertices, ( $N_v = 10$  in the case of the part of Fig. 2).

The total extent area is calculated as the smallest rectangle that can contain a particular nest or tile. This is illustrated in Fig. 3. This total area extent only involves the knowledge of the maximum and minimum x,y values for the parts within the nest or tile. The partial area extent illustrated in Fig. 4, is a more accurate measure of the actual area used by the individual parts within the nest or tile.

This partial extent is calculated by first computing the areas of the smallest rectangles encompassing individual parts and subtracting the "intersecting" area between the parts being grouped into the nest or tile. It is a smaller area than the total extent and it points more directly to a measure of part grouping compactness.

The two extents described above are used through out the various steps in the computer program as the two heuristic variables to determine the effectiveness of a particular nest or primary tile. Weighting factors are assigned as inputs to the computer program for these two extents so as to prefer one more to the other in the calculation. For the illustrated embodiment, both weights are 1.0 (but other values may be used) for nest and primary tile culling.

As an example of how these extents are computed, the following algorithms show these calculations for a nest made up of left and right parts (subscripts "l" and "r"). Here, min,max corresponds to the minimum, maximum coordinate a given part:

10

$$TotExt = [\max(x_{max_l}, x_{max_r}) - \min(x_{min_l}, x_{min_r})] * [\max(y_{max_l}, y_{max_r}) - \min(y_{min_l}, y_{min_r})]$$

$$Ext_l = (x_{max_l} - x_{min_l}) * (y_{max_l} - y_{min_l})$$

$$Ext_r = (x_{max_r} - x_{min_r}) * (y_{max_r} - y_{min_r})$$

15

$$PartExt = Ext_l + Ext_r - [x_{max_l} - x_{min_r}] * [\min(y_{max_l}, y_{max_r}) - \max(y_{min_l}, y_{min_r})]$$

### Basic Structures

The computer program contains five storage structures, where all the data for the various characteristics of the parts, nests, tiles and layouts are accumulated. They are the /Part/, /Nest/, /Ptile/, /STile/ structures (denoted symbolically surrounded by slashes "/"). These variables are more completely described below. For reference, the variables within each structure are now briefly described:

#### /Part/

25 /Part/ contains variables for the storage of the following part characteristics:

1. Part name which correspond to the text file which stores the part's data
2. Number of vertices -  $N_v$
3. The x-y coordinates for the minimum and maximum values-  $X_{min}$  ...  
 $Y_{max}$
- 30 4. The X-Y arrays for the x-y coordinates of dimension  $N_v$

**/Nest/**

/Nest/ contains variables for the storage of the following nest characteristics, each of which is dimensioned to (100) for the maximum number of possible nest that could be generated:

- 5           1.     Left,Right part's side number - nsidel, nsider (an integer between 1-4)
2.     The right part's displacement relative to the left part to form the nest - dxn,dyn
3.     The total and partial extent for the nest - TotExtn,PartExten
4.     The ranked sequence from best to worst nest - Seqncn
- 10          5.     The total number of nests generated by the nesting step (not an array) - Nn

**/PTile/**

/PTile/ contains variables for storage of the following Primary Tile characteristics, each of which is dimensioned to (50,10) for the maximum number of possible nests that are allowed to pass to the primary tiling step = 50, and the maximum number of ptils generated per each nest passed = 10.

1.     The right nest's displacement relative to the left nest, to form the primary tile - dxp,dyp
- 20          2.     The total and partial extent for the ptile - TotExtp,PartExtp
3.     The ranked sequence from best to worst ptile for both the nest passed and the ptile for that nest (singly dimensioned to (50) - Nseqncp,TSeqncp
4.     The total number of ptils generated by the primary tiling step for each nest passed to it (singly dimensioned to (50)) - Np
- 25

**/Stile/**

/Stile/ contains variables for the storage of the following Secondary Tile characteristics, each of which is dimensioned to (50,1080) for the maximum number of possible ptils that are allowed to pass to the secondary tiling step = 50, and the maximum number of stiles generated per each ptile passed = 1080.



1. The top ptile's displacement relative to the bottom ptile, to form the secondary tile dxs,dys
2. The effective parallelogram area used per part - Pa
3. The ranked sequence from best to worst stile for both the ptile passed and the stile for that ptile(singly dimensioned to (50)) - Nseqncs,TSeqncs
4. The total number of stiles generated by the secondary tiling step for each ptile passed to it (singly dimensioned to (50)) - Ns

#### **/Layout/**

/Layout/ contains variables for the storage of the following Layout characteristics.

1. The nest displacement for the best layout - dxlayn,dylayn
2. The ptile displacement for the best layout - dxlayp,dylayp
3. The stile displacement for the best layout - dxlays,dylays
4. The smallest effective parallelogram area used per part, in the final layout - Pamin

#### **Nesting**

Nesting is the process by which two irregular objects or parts are taken and brought together until they "touch" each other on their periphery. This grouping of two parts is called a "nest". A nest is typically formed from two different parts, but for the pictorial examples that follow, the same part is used. The parts are brought towards each other as close together as possible to form a nest as compact as possible.

First, the parts to be nested are circumscribed inside the smallest possible rectangle. The four lateral faces of this rectangle are numbered from 1 to 4. A loop is set up to go through all possible combinations for each face of the left part to be approached by each face of the right part, a total of the combinations in all. Figures 5A and 5B illustrate two nest approaches where Fig. 5A shows nesting for sides 1

and 2 (left and right part respectively) and Fig. 5B shows sides 1 and 1.

Nesting is accomplished in two steps: the axial move (in the x direction or direction of approach), and the tangential move for further reduction of the overall area of the nest. The left part of the nest remains static whereas the right part is  
5 iteratively moved towards the left part in search for the maximum approach to it. Since the approach is in the x direction, a grid is first defined in the y direction which contains the starting locations for the right part's approach toward the left part. A total of ten grid locations is implemented in the y direction. Combining these y grid points with the number of face to face combinations, the maximum number of nests  
10 is generated by the program, which is 100.

The axial move is accomplished by calculating  $N_v$  intersection distances from the left part's vertices towards the right part's sides, and  $N_v$  intersection distances from the right part's vertices toward the left part's sides. The amount that the right part can now be moved towards the left one is the smallest of the  $2-N_v$  distances  
15 calculated for each of the 100 (maximum) nests to be computed.

The tangential move is an adjustment that is implemented once the axial move has been performed. For the tangential move, the part displaced with the intersecting vertex along the other part's intersecting side in the direction along this side until another collision occurs forming a more compact nest. In order to  
20 accomplish this move, the parts are rotated by an angle such that the intersecting side is horizontal, allowing use of the same intersection routine as for the axial move, saving a great deal of computation time.

Figs. 6A-6E show this nesting procedure for the exemplary part of Fig. 2. Fig 6A shows the set up before the axial move. Fig. 6B shows the geometry after the  
25 axial move. Fig. 6C shows the situation after rotation is done so as to render the tangential direction horizontal. At this point, a similar intersection algorithm procedure to the axial move is performed in order to obtain the smallest intersecting line to further approach the right part towards the left one. Fig. 6D depicts the set up after the right part has been moved, and finally; Fig. 6E shows the final nested  
30 situation with the frame of reference rotated back to the original position.

Once the nests are formed, their total and partial area extents are calculated and stored appropriately. Then, these nests are sorted and sequenced, from best to

worst, by seeking for the smallest area extents. Weight factors are applied to each of the two heuristic variables in order to investigate their individual effectiveness. This process is referred to as Nest Ranking.

## 5 Primary Tiling

Once the nests are formed and ranked, a predetermined number of these nests (the ones that ranked highest) are passed to the primary tiling step. In this step, two nests are brought together in the x direction to form a primary tile or "ptile". A ptile is made of two nests or four parts. The same rationale is used to form a ptile as to form a nest. The primary tiling procedure starts by placing two nests side by side in the x direction. As in the nesting step, the left nest remains static whereas the right nest is moved towards the left one as much as possible.

A ten point grid is formed in the y direction for the iterative. Since the maximum number of nests passed to the primary tiling step is 50, a maximum of 500 ptilers are generated. Once the two nests are set up and made ready for axial compression (in the x direction), the intersection algorithm is recalled in order to calculate the smallest distance to approach the right nest towards the left one until any part on the right nest touches any part on the left nest. The parts set up in this process is done in the following manner:

20

- Part 1: Left Part - Left Nest
- Part 2: Right Part - Left Nest
- Part 3: Left Part - Right Nest
- Part 4: Right Part - Right Nest

25

The intersection algorithm routine is then performed four times in sequence to check for the smallest axial approach. The routine is called for parts 1 and 3, then for parts 1 and 4, then for parts 2 and 3, and finally for parts 2 and 4. This allows investigating of the collision of any of the parts on the left nest with any of the parts on the right nest. Fig. 7 shows first the set up situation for a particular primary tile of the exemplary part and then the actual tile achieved.

30

## Secondary Tiling

A small percentage of the ptiles formed (a predetermined number) are then passed onto the secondary tiling step. In the secondary tiling step, as shown in Fig. 8, two ptiles are brought together in the y direction to form a secondary tile or "stile".

5 An stile is made up of two ptiles or four nests or eight parts. In order to set up the geometry for secondary tiling, first two ptiles are placed on the bottom of the domain. Then one more nest is set up along the top that will be nested downwards towards the two ptiles near the bottom of the space. The two ptiles on the bottom are composed of parts numbered 1 through 8 from left to right. The two parts making up the nest at  
10 the top are labeled 9 and 10. The top nest (the one to be dynamically moved to form the stile) is positioned according to whether the ptile has downward or upward flow ( $dyp < 0$  or  $dyp > 0$ ).

Fig. 8 describes the geometry prior to the secondary tiling iteration for the case of  $dyp > 0$ . In that Figure the top nest is positioned just above the bottom left  
15 three nests and to the left of the bottom rightmost nest. The top nest is then iterated on a x grid in a similar manner as the ptile process, except that the x displacement for this grid (as shown in the figure) is equal to the primary tile period. Again, the same rationale as before is used and in the process, ten different stiles are generated for each of the ptiles passed onto this step from the primary tiling step.

20 In order to do a y direction secondary tiling, the coordinate system is first rotated 90 degrees and the previously described axial (x direction) intersection algorithm is performed. In this case, the smallest approach distance between any of the two top parts (parts no. 9 and 10) and any of the six bottom parts is detected (parts no. 1 through 6 for the  $dyp > 0$  case). In the case of  $dyp < 0$ , the top nest is set up  
25 just to the right of the bottom leftmost nest. As a result, the intersection test is made between parts 9 and 10 (top nest) and parts 3 through 8 (bottom tiles).

## Layout

Once the secondary tiling is performed, two numbers are known: the primary  
30 and the secondary tiling period. These are the numbers that represent the displacements in the primary and secondary tiling directions, which enable replication of as many nests as desired on a skewed plane formation for these parts.

Finally, a simple layout routine takes the data generated for the best secondary tile and computes the ideal parallelogram area needed to contain a single nest. This parallelogram area is calculated using the coordinates for the two displacements necessary to form a layout: the primary tiling period ( $x_p, y_p$ ), and the secondary tiling period ( $x_s, y_s$ ). The area of the parallelogram forms is then calculated from the following formulae:

$$\begin{aligned} \ell_p &= \sqrt{x_p^2 + y_p^2} \\ \ell_s &= \sqrt{x_s^2 + y_s^2} \\ \cos \theta_{ps} &= \frac{x_p x_s + y_p y_s}{\ell_p \ell_s} \\ A &= \ell_p \ell_s \sin \theta_{ps} \end{aligned}$$

A computation of the utilization ratio follows by calculating the actual part area and dividing that by the needed parallelogram area. Then, the layout routine creates a file with the CAD commands that generate the best layout for the parts nested.

With the invention, very efficient layouts can be determined with reasonable computation times, using two heuristical variables to eliminate unnecessary branches in the computation process. For simple parts, around 20 nests and primary tiles can be used for passage to higher tiling stages, and still achieve near optimal packing efficiencies. Other numbers of nests and primary tiles can also be used.

Figs. 9 and 10 show exemplary nesting layouts for components of a shoe, which were determined using the invention.

The invention may be embodied in other specific forms without departing from the spirit or essential characteristics thereof. The present embodiments are

therefore to be considered in all respects as illustrative and not restrictive, the scope of the invention being indicated by the appended claims rather than by the foregoing description, and all changes which come within the meaning and range of equivalency of the claims are therefore intended to be embraced therein.

5

## **II. METHOD AND SYSTEM FOR NESTING THREE DIMENSIONAL OBJECTS IN A TARGET VOLUME**

### **REFERENCE TO RELATED APPLICATION**

10 This application is a non-provisional application based on provisional U.S. application Serial No. 60/069,561, filed on December 12, 1997. That application is incorporated by reference herein.

### **FIELD OF THE INVENTION**

15 The present invention is in the field of manufacturing packaging and design and more particularly relates to methods and systems for nesting (or packaging) of three dimensional objects in a target volume. Nesting is the process of placing objects together with minimal space between them.

### **BACKGROUND OF THE INVENTION**

20 There are many design and manufacturing processes in which substantially two dimensional parts-to-be-assembled are initially laid out on and then cut from planes or sheet stock. For example, leather shoes, clothing, pocketbooks, and automobile and furniture upholstery are often formed from a plurality of panels, or parts, cut from a cowhide. After cutting of the panels for such an article,  
25 conventional seam joining techniques, such as sewing, are used to assemble the panels to form the finished article. Clothing, upholstery and many other articles of other materials are similarly designed and manufactured in a similar manner. Such other articles include body parts for automobiles, for example, hoods or fenders,  
30 which may be stamped from metal sheets, and then subsequently shaped to form the final part configuration. Numerous other articles are similarly manufactured from planar sheets as well.

In the manufacture of all of these parts, it is important to minimize the waste which remains after the parts are cut from the sheet stock. While in some cases, where the waste can be re-used, such as in the case of sheet metal, this factor is of lesser importance, there are many cases where the minimization of waste is of critical importance to the commercial viability of a product. The latter cases particularly include shoe manufacture, where the cost of materials, for example cowhides or synthetic materials, is very high.

In view of such cost considerations, manufacturing processes generally include attempts to minimize waste. However, the conventional attempts at such minimization have been relatively ineffective. In the leather shoe industry, for example, parts are often manually die cut from cowhides, where the die is manually placed on the hide by skilled (or relatively unskilled) workers, typically resulting in maximum yields in the 50-60% range for the typical irregularly shaped parts. In recent years, computer controlled layout of parts on a hide have been achieved, resulting in typical yields as high as the 60-70% range.

In U.S. Patent No. 4,210,041, the present inventors disclosed a method for laying out, or nesting, a plurality of identical irregular non-polygonal rounded pieces on a parallel edge-bounded sheet material with relatively little waste. In accordance with the method of that patent, the salient points of the part boundary are connected by straight lines to box the part boundary in a plane polygonal figure. Then, pairs of such plane polygonal figures are pairwise positioned to within a plane closed parallelogram. Finally, a plurality of such parallelogram are arranged side-by-side to define a multiple repeat pattern of objects equal in width to the planar sheet from which the parts are to be cut. A nest constructed in accordance with U.S. Patent 4,210,041 does offer improvement in nesting over prior art techniques; however, the resulting nests are still less than optimal.

Part I of the disclosure as set forth above discloses improvements in nesting of two dimensional objects. That application is incorporated by reference herein.

Further, there is a need to nest three dimensional objects in a target volume. Typically three dimensional objects are manually, or by machine, packed in target volumes with less than optimal techniques. It would be advantageous in many industries to use an optimal technique. For example in the container shipping

industry, containers are packed with objects to be shipped using manually determined placement of the object, generally starting from one border of the container and adding objects one at a time until the container is "full". There are of course cases where an optimized process is easily realized, such as where identical cubic objects are placed in a container having dimensions which are integer multiples of the dimensions of each side of the cubic object. Generally, however, the cubic shape and those container dimension constraints are not present and the conventional packaging methods result in a "filled" container with substantial void regions between the packed objects.

It is an object of the present invention to provide an improved method and system for defining a nest of three dimensional objects.

It is another object to provide an improved method and apparatus for packaging a plurality of three dimensional parts in a target volume.

## SUMMARY OF THE INVENTION

The invention is a method and system for defining placement of a plurality of three-dimensional (3D) objects in a target volume with minimal void space between them. The invention automatically packs the maximum number of irregular parts possible (preferably, of up to two different shapes) with minimum wasted interpart void regions. The invention uses a heuristic approach, so that intelligent decisions are made in order to carry only a small percentage of intermediate solutions to completion. In typical cases, this enables the computation time used in the nesting process to be reduced from hours to minutes, allowing for a more interactive nesting design environment.

The invention uses a combination of topology analysis and numerical iteration to determine an efficient layout in short computation times. The process commences by defining a first sub-nest in a central part of the target volume, grouping parts to each other, and then "grows" or extends outward toward the boundaries of the target volume. The resultant packing layouts are generally parallelogramic, angled in nature. The "angled" layout is contrary to conventional rectangular (90 degree) nesting practice. A heuristic approach is used with an interactive process to curtail long computation times and to produce nested packing



layouts much faster.

The nesting method and system of the invention maximizes target volume utilization from the "center outwards" by minimizing the void regions between nested parts. In contrast, conventional nesting practice first defines a target area, and then fits parts starting from one boundary of the target volume and "flows" parts across the target volume until the opposite edge is reached.

The heuristics used to curtail long computation times, are intermediate measures of how good a nest is. As the process proceeds from one stage to the next, measures of nesting effectiveness are calculated and only a small percentage of all the possibilities are allowed to continue to higher stages. This "nest culling" heuristical process is effective in substantially reducing computation times. By allowing only the best intermediate nests to proceed to primary, secondary and tertiary "tiling" stages, layout times are typically reduced with no practical loss in nesting efficiency.

## BRIEF DESCRIPTION OF THE DRAWINGS

The foregoing and other object of this invention, the various features thereof, as well as the invention itself, may be more fully understood from the following description, when read together with the accompanying drawings in which:

- Fig. 11 shows a block diagram of a system in accordance with the invention;
- Fig. 12A and 12B show an exemplary part to be nested;
- Fig. 13 shows the total nest extent for a nest of the part of Fig. 12;
- Fig. 14 shows the partial nest extent for a nest of the part of Fig. 12;
- Figs. 15A and 5B shows two nest approaches for a nest of the part of Fig. 12;
- Figs. 16A-16E show the nesting procedure for the part of Fig. 12;
- Figs. 17A and 17B show two "better" nests of the part of Fig. 12;
- Fig. 18A and 18B show a set up and actual tile respectively for a primary tile for nests of the part of Fig. 12;
- Fig. 19 shows the geometry prior to secondary tiling for the part of Fig. 12;
- Fig. 20 illustrates the secondary tiling step for the part of Fig. 12; and
- Figs. 21, 22 and 23 show final 3D layouts for the part of Fig. 12.

## DESCRIPTION OF THE PREFERRED EMBODIMENT

The invention is a method and system of nesting a plurality of irregular three dimensional parts into a target volume. The method (and system) uses a combination of topology analysis and numerical iteration to find an efficient layout in short computation times. It starts from an interior (and preferable "center") of the target volume, grouping parts to each other and then "grows" or extends outwards toward the boundaries of the target volume. The resultant packaging layouts are parallelepiped like in nature. This "angled" layout approach is opposite to conventional rectangular (90 degree) nesting practice. A heuristic approach is used in order to curtail long computation times and to rapidly produce nested layouts, giving a designer a much greater interactive responses in his task.

The method and system of the invention maximizes volume utilization from the "center outwards" by minimizing the voids between nested parts.

The heuristics used in order to curtail long computation times, are intermediate measures of how good a nest is. As the computation proceeds from one stage to the next, these measures of nesting effectiveness are calculated and only a small percentage of all the possibilities are allowed to continue to higher stages. This "nest culling" heuristical process substantially reduces computation times. By allowing only the best intermediate nests to proceed to tiling stages, layout times are reduced typically from hours of computation to minutes, with no practical loss in nesting efficiency.

A system 110 embodying the invention is shown in Fig. 11. The system 110 includes a part input device 112 a programmed digital computer 114 with an associated display 116 and an output layout buffer 118. The part input device 112 may be a conventional computer running a computer aided design program, such as AutoCad. The computer may be an IBM type Pentium-based or an Apple PowerPC-based personal computer (with a conventional monitor for display 116) programmed to perform the process of the invention. The layout buffer 118 may include a conventional storage medium for storing the layouts determined by the programmed digital computer 112.

The computer program for computer 110 nests many irregular 3D shapes into

a regularly shaped "box". In defining that program, there are four tasks which are listed below:

**Objective I:** Three dimensional extension of the nesting algorithms of Part I of this disclosure as set forth above.

**Task 1:** Define and implement input of the geometry of parts (components) and targets boxes in three dimensions.

**Task 2:** Extend our main intersection algorithm to three dimensional domain.

**Task 3:** Add three dimensional numerical iteration loops for the part's depth.

**Task 4:** Change measurement of nesting success to void volumes and nest extent volume as opposed to areas."

An automatic three dimensional packaging method and system computes the geometric layout of the various components within the target "box" in an efficient and relatively fast turnaround time. The method and system accomplishes three dimensional nesting in reasonable times and obtains high levels of packing densities.

**Task 1:**

In Task 1, parts or components to be packed and target boxes are defined and input in three dimensional space. Initially, a two dimensional peripheral polygon for the cross-section of the three dimensional object is defined and stored in terms of a list of x, y coordinates for the vertices of the boundary of the part's cross-section. The part's cross-sections are defined as multisided polygons. These polygons can typically have 10's or 100's of sides depending on the complexity of the figure (curvature of its sides) and the

required accuracy of the nesting solution. An extrusion coordinate for every x, y coordinate of this cross-section  $z(x, y)$  defines a large class of solid components, those which have box like appearance.

Nevertheless, not all components may be modeled in this fashion. For example, spheres can not be modeled with this simple extrusion method. Consequently, in those cases a more general method is used that handles all component shapes. In this case, a polygon is first defined for the maximum cross-section. Then this cross-section polygon is subdivided into an (x, y) grid space of approximately ten subdivisions for each (depending on accuracy desired). Finally, left and right extrusion values are defined for each grid node. This more complicated description of the component is particularly useful for complex parts.

A "Multiple Extrusion" solid molding technique is used. This technique defines a series of cross-sectional polygons in the xy plane (typically 5 to 20 cross-sections), which are then multiply extruded/connected in the z direction. Each cross-sectional polygon can be made of (typically) 10 to 100 vertices described by x, y coordinate pairs. In addition each cross-section is associated with a given z coordinate level or value. After connecting corresponding vertices across "z cross-section levels" a three-dimensional body fitted mesh is formed for the peripheral surface of the parts. This method for part input enables a description of a wide variety of 3D solids.

#### **Task 2:**

In Task 2, the intersection algorithm of the above described two dimensional form of the invention is extended into three dimensions.

In the three dimensional case of the invention, coordinates of the form (x, y, z) are used so that a check for minimum translational intersection distance is:

$$\sqrt{(x_2^2 - x_1^2) + (y_2^2 - y_1^2) + (z_2^2 - z_1^2)}$$

5 This check may be done between every "peripheral node" of component 1 with every "peripheral node" of part 2. Once that distance is obtained, it is subtracted from every node on one of the parts into what constitutes the translation of that part toward the other one until they effectively touch each other. The procedure calculates the intersection of part 1 vertices with part 2 sides and part 2 vertices with  
10 part 1 side, for each of overlapping cross-section planes in the z direction. The computation of vertex to side distances is described more fully below in connection with the nesting section of the program description.

### Task 3:

15 The invention uses not only "primary" and "secondary" moves in the nesting and tiling steps, but also a "tertiary" move in the component depth direction. A third nested loop is used for numerical iteration in order to apply the three dimensional intersection algorithm in the "z" direction. This third loop is used at every step of the calculational procedure Nesting, Primary Tiling and Secondary Tiling. In addition a  
20 tertiary tiling step is used in order to place nests and ptils into the z direction. This step is described more fully below in connection with the computer program description.

### Task 4:

25 Geometrical computations are used to determine the size of the nest volume "Total Volume Extent". Obtaining the volume of an assembly of components is a straight forward calculation. The volumetric extent of the void spaces between the components may be computed. However, in the preferred form, Instead of computing nest's void volumes, the "Partial Volume Extent" of a nest is calculated.  
30 As described in more detail, in the nesting section of the computer program

description, this partial extent includes a measure of the volume intersected between parts when forming a nest. This volume saved is the inverse or complement of the void volume used as a heuristic. This "intersection" saved volume is much easier to calculate, and has similar bearing in terms of the effectiveness of a nest as the wasted void volume.

Heuristic algorithms are used for nest culling, total volume extent and partial volume extent. These heuristics limit the number of nests passed to the primary tiling stage and the passage of the primary tiling to second tiling stage. This further limits the search tree and has dramatically improved the performance of the expensive (computer timewise) tiling stage. Many different values for the number of nesting and tiling steps kept and weighting factors may be used for the main heuristic variables. The weighting factors for both the heuristical variables (total and partial volume extents) are constant and set to 1.0, as an example. Other numbers might be used.

#### COMPUTER PROGRAM DESCRIPTION

This section provides a brief description of the computer program used in computer 114. The hardware platform is an Apple Macintosh computer with a 17" color monitor and laser printer. The CAD software used for this embodiment consists of two packages- Autodesk's AutoCAD and Graphsoft's MiniCAD.

This embodiment was developed using the following combination of software tools: 1- Macintosh Programmer's Workshop (MPW) for the operating environment(including editing and debugging); 2- FORTRAN Compiler for the compilation of the program; and 3- MiniCAD for the three-dimensional part input and output layout visualization. The CAD package permits description of the 3D parts using the "Multiple Extrusion" method and interactive visualization of the 3D layouts. AutoCAD's AME solid modeling could alternatively be used to provide a more complete 3D solids description but with a more complex numerical description.

The inner regions of the nest have great importance in the invention (as

opposed to the peripheral or edge regions near the target's boundaries). The computation is segmented into four sequential steps: Nesting, Primary Tiling, Secondary Tiling, and Tertiary Tiling/Layout. The process is initiated with the **nesting** step, two irregular 3D objects or parts are taken and "brought together" until they "touch" each other on their periphery. This grouping of two parts is called a "nest". These two parts are brought towards each other, as close together as possible to form as compact a nest as possible. Depending on the complexity of the parts being nested, the computer program performs from many hundreds to many thousand different nests and ranks them according to the compactness (using heuristic variables as previously described).

Once the nests are formed and ranked, a predetermined number of those nests (the ones that ranked highest) are passed to the **primary tiling** step. In this step, two nests are brought together in the x direction to form a primary tile or "ptile". A ptile is made of two nests or four parts. The same rational is used to form a ptile as to form a nest. That is, a grouping of two nests is created that forms the most possible compact unit. Typically, a few hundred ptils are generated per each nest passed from the nesting step. These ptils are consequently ranked using heuristic variables for compactness.

As small percentage of the ptils formed (a predetermined number) are then passed onto the **secondary tiling** step. In this case, two ptils are brought together in the y direction to form a secondary tile or "stile". A stile is made up of two ptils or four nests or eight parts. Again, the same rational as used for nest formation is used, an in the process, over a thousand different stiles are generated for each of the ptils passed onto this step from the primary tiling step. For each of these secondary tiles produced, a **tertiary tiling** is implemented. In the tertiary tiling secondary tiles are brought together in the z direction, using a simple stacking rational. The stiles are brought only as close as their closest boundaries, without optimization; that step is performed for every stile generated without secondary tile culling. In other embodiments, stiles are nested with appropriate culling to reduce computational time.

Finally, a **layout** routine takes the data generated for the best tertiary tile and

computes the ideal parallelepiped volume needed to contain a single nest. A computation of the utilization ratio follows by calculating the actual part volume and dividing that by the needed parallelepiped volume. Then the layout routine creates a file with the 3D CAD commands that generate the best layout for the parts nested.

5 That layout may be displayed on monitor 116, or stored and/or printed in layout buffer/printer 118.

The following provides a step by step description of the computer program, starting with the method for describing 3D parts and ending with a description of the computation steps: Nesting, Tiling and Layout.

10

### Three Dimensional Part Description

In order to describe and input parts in three dimensions, the program uses a "Solid Modeling" technique called "Multiple Extrusion". Using this technique, a series of cross-sectional polygons are defined in the xy plane (typically 5 to 20 cross-  
15 sections), which are then multiply extruded/connected in the z direction. Each cross-sectional polygon can be made of (typically) 10 to 100 vertices described by x, y coordinate pairs. In addition, each cross-section is associated with a given z coordinate level, or value. After connecting corresponding vertices across "z cross-section levels", a three-dimensional body fitted mesh is formed for the peripheral  
20 surface of the parts. This method for part input enables description of a wide variety of 3D solids including irregular boxes and spherical objects.

As an example of this methodology, Figs. 12A and 12B shows a 3D object  
140 created with four cross-sectional z levels labeled  $z=0, 0.4, 0.8$  and  $1.2$ . These are the values of the z coordinate at each cross-sectional level. Each cross-section in turn  
25 lies on the xy plane and is composed of ten vertices labeled with coordinates  $(x_1, x_2)$  through  $(x_{10}, y_{10})$ . This 3D figure is thus fully described with four sets of ten x, y coordinate pairs for a total of eighty numbers. This particular figure of Figs. 12A and 12B is used in the following illustration of the invention.

The data defining the object 140 is stored in a text file, with all the x, y  
30 coordinate pairs. Thus, the invention enables a relatively straight forward design and input of a large variety of 3D parts using this multiple extrusion model.



### Part Volume Calculation

Three different volumes are calculated: 1- the actual part volume, 2- the total volume extent for nests and tiles, and 3- the partial volume extent for nests and tiles. The computation of the part volume is relatively straightforward: the quadrilateral faces formed by the four vertices across two z levels are defined to be straight two-dimensional planes. If  $dz$  the z coordinate interval between cross-sections is referred to as  $dz$  and  $y_{\min}$  is defined as the smallest y coordinate at each cross-section, then the cross-section areas  $A_k$  and part volume  $V$  are:

$$A_k = \sum_{j=0}^{N_v} \frac{1}{2} (X_{i+1} - X_i) (Y_{i+1} + Y_i - 2Y_{\min})$$

$$V = \sum_{k=1}^{N_z} \frac{1}{2} (A_{k+1} + A_k) dz$$

where  $N_v$  and  $N_z$  are the number of vertices per each cross-section, and the total number of cross-sections, respectively (10 and 4 in the case of object 140).

The total extent volume is the smallest box that can contain a particular nest or tile. This is illustrated on in Fig. 15. This total volume extent is calculated from the knowledge of the maximum and minimum x, y, z values for the parts within the nest or tile. The partial volume extent is illustrated in Fig. 16. That value is an accurate measure of the actual volume used by the individual parts within the nest or tile.

The partial extent is calculated by first computing the volumes of the smallest boxes encompassing individual parts and subtracting the "intersecting" volume between the parts being grouped into the nest or tile. It is a smaller volume than the "total extent" and it points more directly to a measure of part grouping compactness.

The total volume extend and partial extent described above are used throughout the various steps in the computer program as the two heuristic variables to determine the effectiveness of a particular nest or primary tile. Weighting factors are assigned as inputs to the computer program for these two extents so as to prefer one more to the other in the calculation. Both weights are set to 1.0 in the illustrated embodiment, but other values for these heuristic may be used for different nest and primary tile culling.

The algorithms for calculation for a nest made up of left and right parts (subscripts "l" and "r") are shown below. In this example,  $ymin_l$  corresponds to the minimum y coordinate for the left part. The total extent (totExt) and partial extent (PartExt) are determined in accordance with:

$$\begin{aligned}
 \text{TotExt} &= [\max(xmax_l, xmax_r) - \min(xmin_l, xmin_r)] * \\
 &\quad [\max(ymax_l, ymax_r) - \min(ymin_l, ymin_r)] * \\
 &\quad [\max(zmax_l, zmax_r) - \min(zmin_l, zmin_r)] \\
 \text{Ext}_l &= (xmax_l - xmin_l) * (ymax_l - ymin_l) * (zmax_l - zmin_l) \\
 \text{Ext}_r &= (xmax_r - xmin_r) * (ymax_r - ymin_r) * (zmax_r - zmin_r) \\
 \text{PartExt} &= \text{Ext}_l + \text{Ext}_r - [xmax_l - xmin_l] * \\
 &\quad [\min(ymax_l, ymax_r) - \max(ymin_l, ymin_r)] * \\
 &\quad [\min(zmax_l, zmax_r) - \max(zmin_l, zmin_r)]
 \end{aligned}$$

### Basic Structures

The computer program contains five storage structures where all the data for the various characteristics of the parts, nests, tiles and layouts are accumulated. Those structures are the /Part/, /Nest/, /Ptile/, /Stile/ and /Layout/ structures (denoted symbolically surrounded by slashes "/"). These variables are more thoroughly described below. The variables within each structure are:

#### /Part/

/Part/ contains variables for the storage of the following part characteristics:

1. Part name which correspond to the text file which stores the part's vertex data
2. Number of vertices per cross-section layer:  $N_v$
3. Number of cross-section layers:  $N_z$
4. The x, y coordinates for the minimum and maximum values:  $x_{min} \dots y_{max}$
5. The z coordinates for the lowest and highest cross-section layer -  $z_{min}, z_{max}$
6. The z layer interval:  $dz$
7. The X, Y arrays for the x, y coordinates of dimension  $(N_v, N_z)$

7. The X, Y arrays for the x, y coordinates of dimension ( $N_x, N_y$ )
8. The Z array for the z coordinates of dimension ( $N_z$ )

**/Nest/**

5 /Nest/ contains variables for the storage of the following nest characteristics, each of which is dimensioned to (7400) for the maximum number of possible nests that could be generated:

1. Left, Right part's side number - nsidel, nsider (an integer between 1-4)
2. A switch variable indicating whether the right part has been flipped in the z  
10 axis
3. The right part's displacement relative to the left part to form the nest:  $dx_n$ ,  
 $dy_n$ ,  $dz_n$
4. The total and partial extent for the nest: TotExt<sub>n</sub>, PartExt<sub>n</sub>
5. The ranked sequence from best to worst nest: Seqnc<sub>n</sub>
- 15 6. The total number of nests generated by the nesting step (not an array) -  $N_n$

**/PTile/**

20 /PTile/ It contains variables for the storage of the following Primary Tile characteristics, each of which is dimensioned to (50,710) for the maximum number of possible nests that are allowed to pass to the primary tiling step = 50, and the maximum number of ptils generated per each nest passed = 710.

1. The right nest's displacement relative to the left nest, to form the primary tile  
-  $dx_p$ ,  $dy_p$ ,  $dz_p$
2. The total and partial extent for the ptil: TotExt<sub>p</sub>, Part Ext<sub>p</sub>
3. The ranked sequence from best to worst ptil for both the nest passed and the  
25 ptil for that nest (singly dimensioned to (50)): NSeqnc<sub>p</sub>, Tseqnc<sub>p</sub>
4. The total number of ptils generated by the primary tiling step for each nest  
passed to it (singly dimensioned to (50)):  $N_p$

**/STile/**

30 /STile/ contains variables for the storage of the following Secondary Tile characteristics, each of which is dimensioned to (50,1080) for the maximum number of possible ptils that are allowed to pass to the secondary tiling step = 50, and the maximum number of stiles generated per each ptil passed = 1080.

1. The top ptile's displacement relative to the bottom ptile, to form the secondary tile:  $dx_s, dy_s, dz_s$
2. The effective parallelepiped volume used per part:  $P_v$
3. The ranked sequence from best to worst stile for both the ptile passed and the  
5 stile for that ptile (singly dimensioned to (50)):  $N_{Seqnc_s}, T_{Seqnc_s}$
4. The total number of stiles generated by the secondary tiling step for each ptile passed to it (singly dimensioned to (50)):  $N_s$

#### /Layout/

10 /Layout/ contains variables for the storage of the following Layout characteristics.

1. The nest displacement for the best layout:  $dx_{lay_n}, dy_{lay_n}, dz_{lay_n}$
2. The ptile displacement for the best layout:  $dx_{lay_p}, dy_{lay_p}, dz_{lay_p}$
3. The stile displacement for the best layout:  $dx_{lay_s}, dy_{lay_s}, dz_{lay_s}$
- 15 4. The z direction displacement for the best layout:  $dx_{lay_z}, dy_{lay_z}, dz_{lay_z}$
5. The smallest effective parallelepiped volume used per part, in the final layout:  
 $P_{v_{min}}$

#### Nesting

20 Nesting is the process by which two irregular 3D objects or parts are taken and brought together until they "touch" each other on their periphery. This grouping of two parts is called a "nest". A nest is typically formed from two different parts, but for the pictorial examples that follow the same part is used. The parts are brought towards each other as close together as possible to form a nest as compact as  
25 possible.

In performing this task in three dimensions, the parts to be nested are first circumscribed inside the smallest possible box. In the illustrated embodiment, the bottom and top z surfaces of the bounding box are not fully nested, but only are stacked; in other embodiments, full nesting (optimal) is performed. The other four  
30 lateral faces of this box are numbered from 1 to 4. The program sets up a loop to go through all possible combinations for each face of the left part to be approached by each face of the right part, a total of ten combinations in all. Another ten face-to-face

combinations are to accommodate when the right part is "flipped" along the z axis, yielding a total of twenty face combinations used for nesting. Figs. 15A and 15B illustrate two nest approaches. Fig. 15A shows nesting for sides 1 and 2 (left and right part respectively) and Fig. 15B shows sides 1 and 1 with the right part flipped.

5 Nesting is accomplished in two steps: (1) an axial move (in the x direction or direction of approach), and (2) a tangential move for the further reduction of the overall volume of the nest. In the illustrated embodiment, the left part of the nest remains static while the right part is iteratively moved towards the left part in search for the maximum approach to it. Since the approach is in the x direction, a grid in  
 10 the yz plane is first defined that grid contains the starting locations for the right part's approach toward the left part. A total of ten grid locations is implemented in the y direction while in the z direction, in increments of the interlevel distance dz are stepped through.

As a result, the total number of yz plane grid points is  $10(n_{zl}+n_{zr}-3)$ . Where  
 15 the maximum number of z cross-section levels allowed per part is 20, there are a maximum of 370 iterative grid points in the yz plane. The number of nests generated (7400) is determined by combining these yz grid points with the number of face to face combinations. For the exemplary object 140, having only 4 cross-sections there are only 50 yz grid points and a total of 1,000 possible nests generated.

20 The axial move is accomplished by calculating  $N_v$  intersection distances from the left part's vertices towards the right part's sides, and  $N_v$  intersection distances from the right part's vertices toward the left part's sides. This is performed on each and every z level cross-section plane, preferably using the basic 2D intersection algorithm disclosed in the incorporated reference. The intersection algorithm is  
 25 executed for every z plane cross-section and for each of the yz plane grid points. The amount that the right part is moved towards the left one is the smallest of the  $2 \cdot N_v \cdot N_z$  distances calculated for each of the 7,400 (maximum) nests to be computed.

The tangential move is an adjustment implemented once the axial move has been performed. For the tangential move, the part is displaced with the intersecting  
 30 vertex along the other part's intersecting side in the direction along that side until another "collision" occurs and a more compact nest is formed. In order to

accomplish that move, parts are "rotated" by an angle such that the intersecting side is horizontal, allowing use of the same intersection routine as for the axial move.

Figs. 16A-16E show the nesting procedure for one of the z cross-section levels. The first step (Fig 16A) shows the set up before the axial move. The second  
5 step (Fig. 16B) shows the geometry after the axial move. The third step (Fig. 16C) shows the geometry after rotation is done so as to render the tangential direction horizontal. At this point, a similar intersection algorithm procedure to the axial move is performed in order to obtain the smallest intersecting line to further approach the  
10 right part towards the left one. The fourth step (Fig. 16D) depicts the set up after the right part has been moved, and finally the fifth step (Fig. 16E) shows the final nested situation with the frame of reference rotated back to the original position.

Once the nests are formed their total and partial volume extents are calculated and stored. Then, a sorting of these nests is performed and the nests are sequenced  
15 from best to worst by seeking for the smallest volume extents. Weight factors are applied to each of the two heuristic variables in order to determine their individual effectiveness. This process is referred to as Nest Ranking. Fig. 17A and 17B show two of the better nests formed with the test object 140.

### Primary Tiling

20 Once the nests are formed and ranked, a predetermined number of the nests (the ones that ranked highest) are passed to the primary tiling step. In this step two nests are brought together in the x direction to form a primary tile or "ptile". A ptile is made of two nests or four parts, and the same rational is used to form a ptile as  
25 used to form a nest. The primary tiling procedure starts by placing two nests side by side in the x direction. As in the nesting step, the left nest remains static, while the right nest is moved towards the left one as much as possible.

A grid is formed in the yz plane for the iterative approach, with ten grid  
30 points in the y direction and a number of z grid points corresponding to all the corresponding intersecting z cross-section levels between the two nests. The maximum number of z grid points  $2(N_{zl}+N_{zr}-3)-3$  is 71. Typically, a few hundred ptiles are generated per each nest passed from the nesting step. As a maximum for

the exemplary computer program (only twenty z levels allowed), there are 710 ptils for every nest. For the test object 140, there are only 70 ptils per nest. Since the maximum number of nests passed to the primary tiling step is set to 50, a maximum of 3,550 ptils are generated, but for test object 140, the maximum number of ptils is 350.

Once the two nests are set up and made ready for axial compression (in the x direction), the intersection algorithm is recalled in order to calculate the smallest distance to approach the right nest towards the left one until any part on the right nest touches any part on the left nest. The computer program numbers the parts set up in this process in the following manner:

Part 1: Left Part - Left Nest

Part 2: Right Part - Left Nest

Part 3: Left Part - Right Nest

Part 4: Right Part - Right Nest

The intersection algorithm routine is then performed four times in sequence to check for the smallest axial approach. The routine is called for parts 1 and 3, then for part 1 and 4, then for parts 2 and 3, and finally for parts 2 and 4. This allows determination of the collision of any of the parts on the left nest with any of the parts on the right nest. Figs. 18A and 18B show first the set up situation for a particular primary tile of test object 140 (Fig. 18A), and then the actual tile achieved (Fig. 18B).

### Secondary Tiling

A small percentage of the ptils formed (a predetermined number) are then passed onto the secondary tiling step. In the secondary tiling step, two ptils are brought together in the y direction to form a secondary tile or "stile". An stile is made up of two ptils or four nests, or eight parts. In order to set up the geometry for secondary tiling, first two ptils are placed on the bottom of the domain. Then one more nest is set up along the top that is to be nested downwards towards the two ptils near the bottom of the space. The two ptils on the bottom are composed of parts numbered 1 through 8 from left to right. The two parts making up the nest at the top, are labeled 9 and 10. The top nest (the one to be dynamically

moved to form the stile) is positioned according to whether the ptile has downward or upward flow ( $dy_p < 0$  or  $dy_p > 0$ ).

Fig. 19 shows geometry prior to the secondary tiling iteration for the case of  $dy_p > 0$ . In that figure, the top nest is positioned just above the bottom left three nests and to the left of the bottom rightmost nest. The top nest is then iterated on a xz grid in a similar manner as done in the ptile process, except that the x displacement for this grid (as shown on the figure) is equal to the primary tile period. Again the same rational as before is used, and in the process over a thousand different stiles are generated for each of the ptiles passed onto this step from the primary tiling step.

In order to accomplish a y direction secondary tiling, the coordinate system is rotated 90 degrees, and the previously described axial (x direction) intersection algorithm is performed. In this case a check is performed to determine the smallest approach distance between any of the two top parts (part nos. 9 and 10) and any of the six bottom parts (part nos. 1 through 6 for the  $dy_p > 0$  case). In the case of  $dy_p < 0$  the top nest is set up just to the right of the bottom leftmost nest. As a result, the intersection test is made between parts 9 and 10 (top nest) and parts 3 through 8 (bottom tiles). The resulting stile is shown in Fig. 20.

### **Tertiary Tiling/Layout**

Performance of the primary and secondary tiling steps yields two numbers: the primary and the secondary tiling period. These numbers represent the displacements in the primary and secondary tiling directions, and replication of as many nests as desired on a skewed plane formation for these parts. For each of these secondary tiles produced, a tertiary tiling is also implemented. In the tertiary tiling process of the illustrated embodiment, secondary tiles are brought together in the z direction using a simple stacking rationale, bringing the stiles no closer than their closest boundaries. In that form, the layout is an inexpensive calculation and can be performed for every stile generated without the need for secondary tile culling. In other forms of the invention, the secondary tiled configuration can be optimally nested into stacks with optimal packing. The latter process can be performed with reduced computational time using the above described heuristic method.



The "stacking" of skewed planes on top of each other is performed by displacing the layers to avoid collisions when nests or tiles have been formed with non-zero z direction displacements.

A simple layout routine takes the data generated for the best tertiary tile and computes the ideal parallelepiped volume needed to contain a single nest. This parallelepiped volume is calculated using the coordinates for the three displacements necessary to form a layout: the primary tiling period ( $x_p, y_p, z_p$ ), the secondary tiling period ( $x_s, y_s, z_s$ ) and the displaced z direction stacking period ( $x_z, y_z, z_z$ ). The volume of the parallelepiped formed is calculated from the following formulae:

$$\begin{aligned}\ell_p &= \sqrt{x_p^2 + y_p^2 + z_p^2} \\ \ell_s &= \sqrt{x_s^2 + y_s^2 + z_s^2} \\ \ell_z &= \sqrt{x_z^2 + y_z^2 + z_z^2} \\ \cos \theta_{ps} &= \frac{x_p x_s + y_p y_s + z_p z_s}{\ell_p \ell_s} \\ \cos \theta_{pz} &= \frac{x_p x_z + y_p y_z + z_p z_z}{\ell_p \ell_z} \\ \cos \theta_{sz} &= \frac{x_s x_z + y_s y_z + z_s z_z}{\ell_s \ell_z} \\ Vol &= \ell_p \ell_s \ell_z \sin \theta_{ps} \sin \theta_{pz} \sin \theta_{sz}\end{aligned}$$

A computation of the utilization ratio is determined by calculating the actual part volume and dividing that volume by the needed parallelepiped volume. Then the layout routine creates a file with the 3DCAD commands that generate the best layout for the parts nested.

## Performance of the Illustrated Embodiment

### Example 1. Layout Results Vs Number of Nests/PTiles Kept

The above defined computer system (Fig. 11) and computer program was implemented for the test object 140 of Fig. 12. The number of nests and ptils kept for passage to further stages of the computation was varied simultaneously between 5

and 50. That is, these two numbers were varied equally ( $N_{n-pt}=N_{pt-st}$ ). A variation from 5 to 50 represents more than two orders of magnitude variation in computation time. The programmed computer produced a layout result for only 5 nests/ptiles kept in less than a minute, whereas it produced a layout for 50 nests/ptile in approximately one hour.

The test object 140 tested is a very simple one with only a few vertices and cross-section levels to describe it. More complex parts may have typically 10 times more vertices and say 5 times more cross-sections to describe it. This fifty-fold increase in computation time means that for a small number of nests/ptiles kept, the computer processing time will now be on the order of an hour, whereas for the large number of nests/ptiles kept, the numerical solution could typically take many hours. Even with the use of faster computers (e.g., a ten fold increase in speed), a long wait would be expected for a 3D layout. The heuristic nest culling process, achieves near optimal layout utilization efficiencies, while using much smaller computation times.

The following table shows volume utilization using the configuration of this example, the number of nests/ptiles kept for the test object 140.

$N_{n-pt}$	$N_{pt-st}$	Percent Volume Utilization
5	5	35%
10	10	41%
20	20	42%
50	50	42%

The data in the above table suggests that, keeping 20 nests is all that is needed in order to achieve near optimal results. More complex parts will also require a larger number of nests/ptiles kept for passage onto higher stages in the computation.

The three layouts produced for the three cases of 5, 10 and 20 nests/ptiles kept are shown in Figs. 21-23. The  $N=10$  case (Fig. 22) used a different nest than the  $N=5$  case (Fig. 21) and yielded an increase in efficiency of 6%. The  $N=20$  case (Fig. 23) kept the same nest used in the  $N=10$  case but used a more efficient ptile (which was not kept within the ten limitation) in order to increase the utilization efficiency another 1%. In order to achieve near optimal volume utilization, the weighting

In other embodiments, the computer program incorporates z face nesting and tiling as well as lateral nesting tiling. The tangential move of the nesting step is used in both the primary and second tiling steps to produce even more compact layouts. A full intersecting tertiary tiling step (as opposed to the simple "semi-stacking" tiling procedure) can also be used for improved layouts. A more complex 3D part description may be used, permitting multiple connected cross-sections into the geometric algorithms of the numerical calculation.

15

What is claimed is:

- 1 Method defining a packing configuration for a plurality of two dimensional (2D) objects in a target plane, comprising the computer generated steps of:
  - 5 A. generating signals defining each of said 2D objects with respect to an object coordinate system having orthogonal axes  $X_0$  and  $Y_0$ , in terms of a plurality of points representative of boundary points on said objects,
  - 10 B. associating two of said objects to define an object pair,
  - C. generating a nest signal representation of said objects of said object pair nested with respect to a nest coordinate system having orthogonal axes  $X_N$  and  $Y_N$ , to define a plurality of close-packed candidate nests in which the pairwise positions of two said objects of each candidate nest are characterized by relatively small planar spaces between the respective objects,
  - 15 D. selecting a subset of said candidate nests in accordance with predetermined nest criteria,
  - E. for each of said selected candidate nests, generating a tile signal representative of two such nests tiled with respect to a tile coordinate system having orthogonal axes  $X_T$  and  $Y_T$ , said tiling steps including the sub steps of:
    - 20 I. generating a primary tile signal representative of said two nests tiled with respect to said  $X_T$  axis by translating said nests with relative motion in the direction of said  $X_T$  and  $Y_T$  axes, to define a plurality of candidate primary tiles in which the pairwise
    - 25
    - 30

positions of said two nests of each candidate primary tile are characterized by relatively small planar spaces between said nests,

- 5           ii.     selecting a subset of said candidate primary tiles in accordance with predetermined primary tile criteria,
- iii.    each of said selected primary tiles, generating a secondary tile signal representative of a nest and said primary tile tiled with respect to said  $Y_T$  axis by translating said nest with respect to said primary tile with relative motion in the direction of said  $X_T$  and  $Y_T$  axes, to define a plurality of candidate secondary tiles in which the pairwise positions of said nest and said primary tile are characterized by relatively small planar spaces between said nest and the nests of said primary tile,
- 10           iv.    selecting one of said candidate secondary tiles in accordance with predetermined secondary tile criteria, thereby defining the optimal nest and primary tile, said optimal nest defining said packing configuration.
- 15
- 20

2.     The method according to claim 1 comprising the further steps of:

- 25           F.     generating an initial tile signal representative of an initial secondary tile positioned in said target plane at a position other than adjacent the boundary of said target plane, and
- 30           G.     generating an area packing signal representative of said initial tile as augmented additional secondary tile extending successively from said initial secondary tile in the direction of each of said  $X_T$ , and  $Y_T$  axes,

whereby adjacent secondary tiles are substantially identically pairwise configured in said target plane in the direction of said  $X_T$  and  $Y_T$  axes.

3. A system for defining a packing configuration for a plurality of two dimensional (2D) objects in a target plane, comprising:
  - A. means for generating signals defining each of said 2D objects with respect to an object coordinate system having orthogonal axes  $X_0$  and  $Y_0$ , in terms of a plurality of points representative of boundary points on said objects,
  - B. means for associating two of said objects to define an object pair,
  - C. means for generating a nest signal representation of said objects of said object pair nested with respect to a nest coordinate system having orthogonal axes  $X_N$  and  $Y_N$ , to define a plurality of close-packed candidate nests in which the pairwise positions of two said objects of each candidate nest are characterized by relatively small planar spaces between the respective objects,
  - D. means for selecting a subset of said candidate nests in accordance with predetermined nest criteria,
  - E. tiling means operative for each of said selected candidate nests, for generating a tile signal representative of two such nests tiled with respect to a tile coordinate system having orthogonal axes  $X_T$  and  $Y_T$ , said tiling means including:
    - i. means for generating a primary tile signal representative of said two nests tiled with respect to said  $X_T$  axis by translating said nests with relative motion in the direction of said  $X_T$  and  $Y_T$  axes, to

5

- 10

15

20

25

- 30

- 5 G. means for generating an area packing signal representative of said initial tile as an augmented additional secondary tile extending successively from said initial secondary tile in the direction of each of said  $X_T$  and  $Y_T$  axes, whereby adjacent secondary tiles are substantially identically pairwise configured in said target plane in the direction of said  $X_T$  and  $Y_T$  axes.
- 10 5. Method of defining a packing configuration for a plurality of three dimensional (3D) objects in a target volume, comprising the computer generated steps of:
- 15 A. generating an object signal defining each of said 3D objects with respect to an object coordinate system having orthogonal axes  $X_0$ ,  $Y_0$  and  $Z_0$ , in terms of a plurality of points representative of boundary points on said objects,
- 20 B. associating two of said objects to define an object pair,
- 25 C. generating a nest signal representative of said objects of said object pair nested with respect to a nest coordinate system having orthogonal axes  $X_N$ ,  $Y_N$  and  $Z_N$ , to define a plurality of close-packed candidate nests in which the pairwise positions of said objects of each candidate nest are characterized by relatively small volumetric spaces between the respective objects,
- 30 D. selecting a subset of said candidate nests in accordance with predetermined nest criteria,
- E. for each of said selected candidate nests, generating a tile signal representative of two of said nests tiled with respect to a tile coordinate system having orthogonal axes  $X_T$ ,  $Y_T$  and  $Z_T$ , said tile generating step including the sub steps of:



- 5

i. generating a primary tile signal representative of said two nests tiled with respect to said  $X_T$  axis by translating said nests with relative motion in the direction of said  $X_T$ ,  $Y_T$  and  $Z_T$  axes, to define a plurality of candidate primary tiles in which the pairwise positions of said two nests of each candidate primary tile are characterized by relatively small volumetric spaces between said nests,
- 10

ii. selecting a subset of said candidate primary tiles in accordance with predetermined primary tile criteria,
- 15

iii. for each of said selected primary tiles, generating a secondary tile signal representative of one of said nests of said primary tiles and said primary tile tiled with respect to said  $Y_T$  axis by translating said nest with respect to said primary tile with relative motion in the direction of said  $X_T$ ,  $Y_T$  and  $Z_T$  axes, to define a plurality of candidate secondary tiles in which the pairwise positions of said nest and said primary tile are characterized by relatively small volumetric spaces between said nest and the nests of said primary tile,
- 20

iv. selecting one of said candidate secondary tiles in accordance with predetermined secondary tile criteria,
- 25

v. for each of said selected secondary tiles, generating a tertiary tile signal representative of two of said secondary tiles tiled with respect to said  $Z_T$  axis by translating said secondary tiles with respect to each other with relative motion of in the direction of said  $X_T$ ,  $Y_T$ , and  $Z_T$  axes to define a plurality of tertiary tiles

30

5

- 10

- 15

- 20

- 25

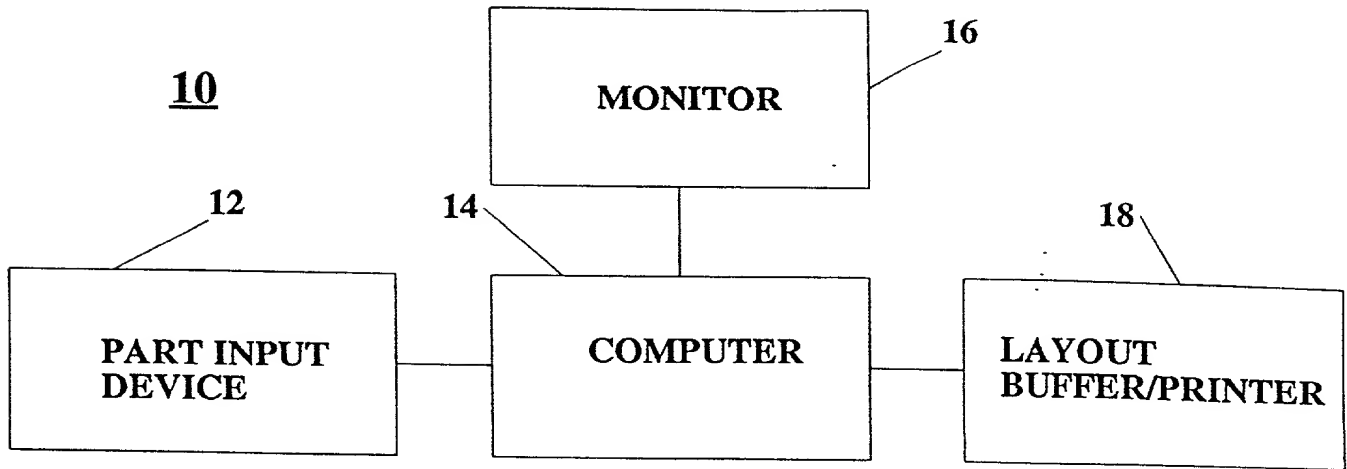


FIG. 1

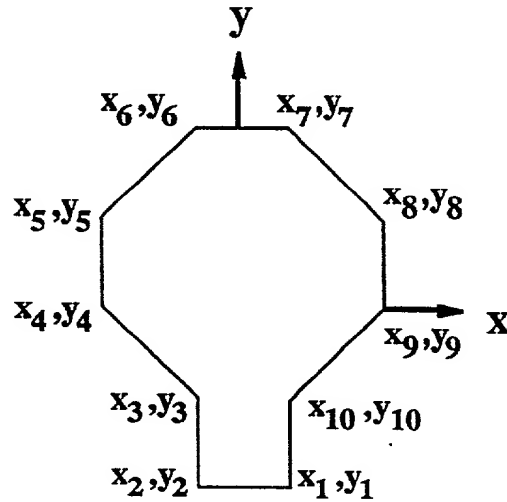


FIG. 2

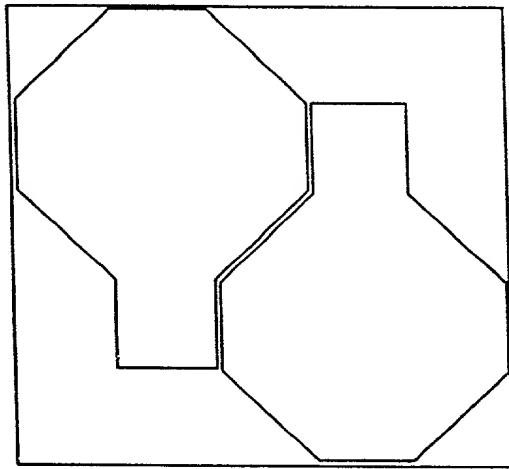


FIG. 3

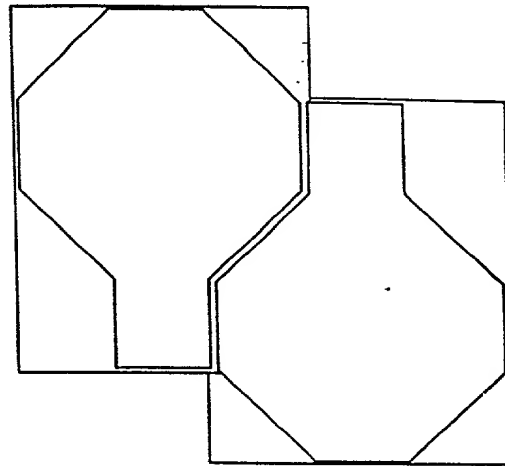


FIG. 4

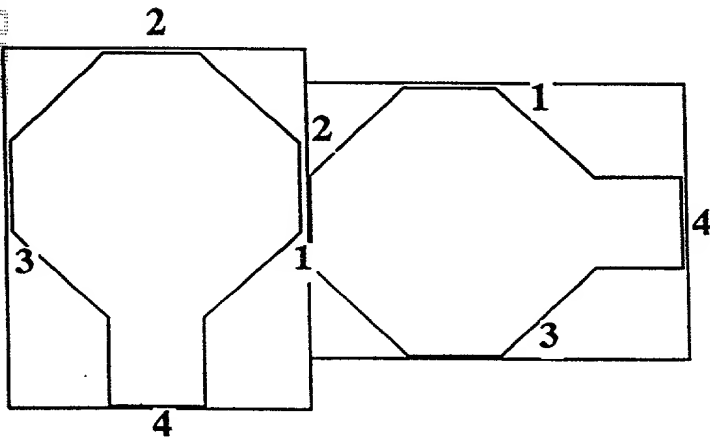


FIG. 5A

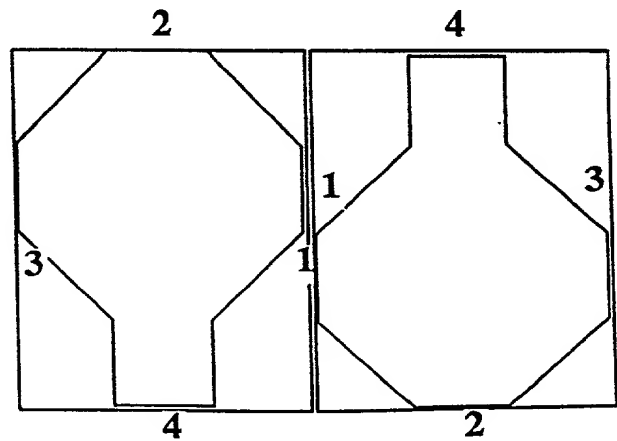


FIG. 5B

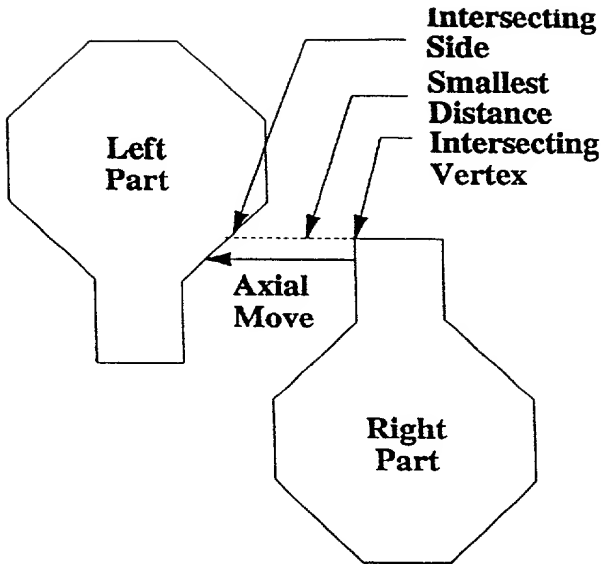


FIG. 6A

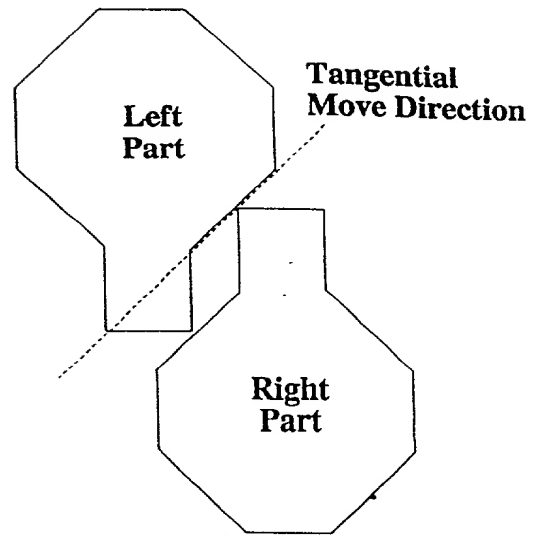


FIG. 6B

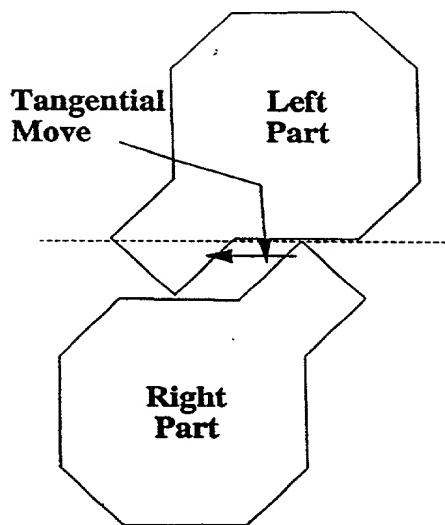


FIG. 6C

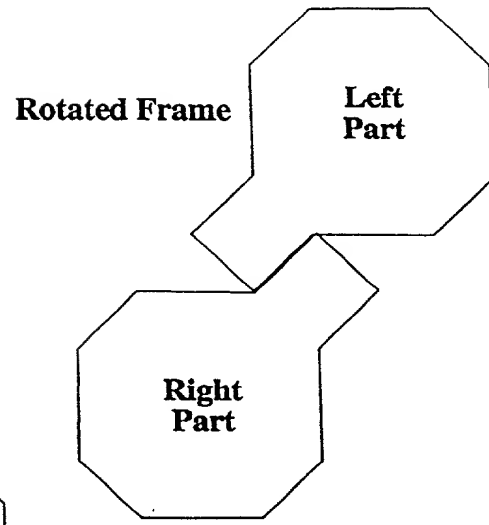


FIG. 6D

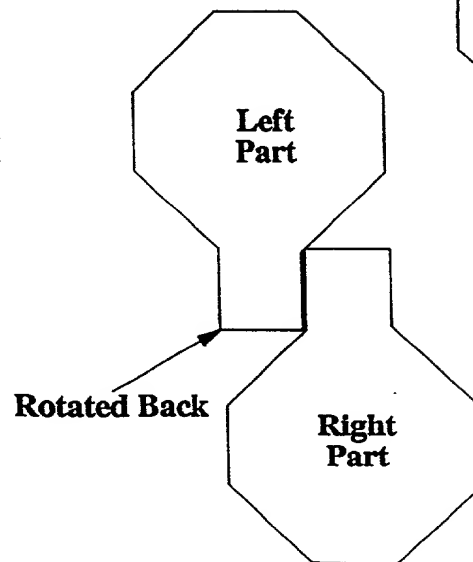


FIG. 6E

008707" 50070500

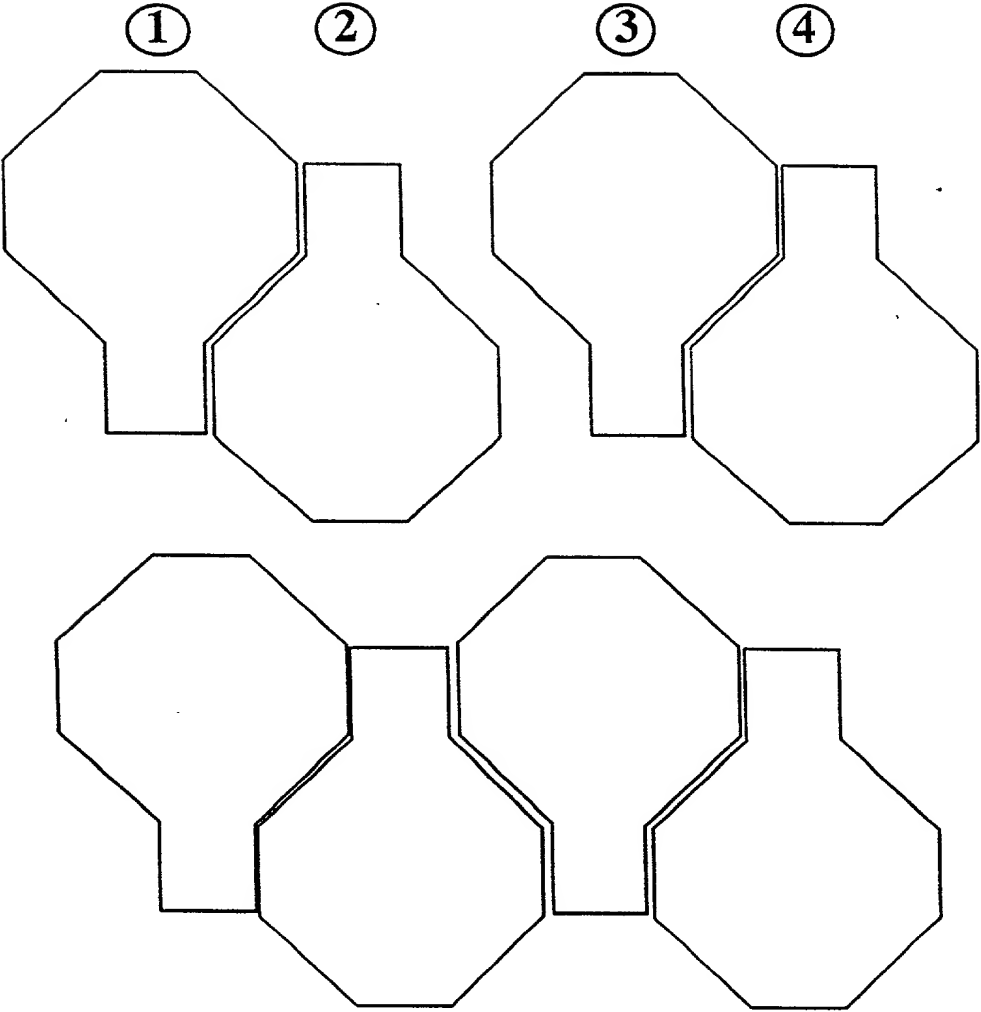


FIG. 7



Ideal Utilization Ratio	=	.8397
RMedEyestay		
Target Utilization Ratio	=	.7336
for a 3m X 1m Sheet		
Target Utiliz for a 5m X 1m	=	.7612

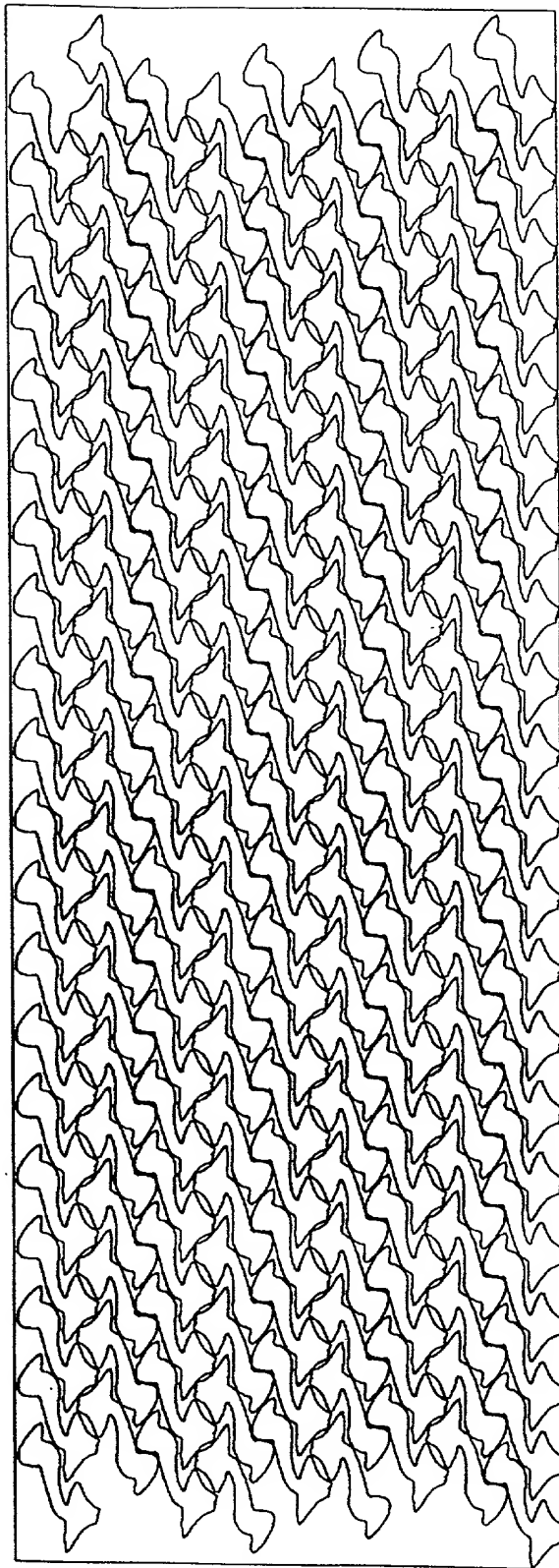


FIG. 9



Ideal Utilization Ratio = .8201  
RBTrip RBTrip  
Target Utilization Ratio = .7813  
for a 3m X 1m Sheet  
Target Utiliz for a 5m X 1m = .7943

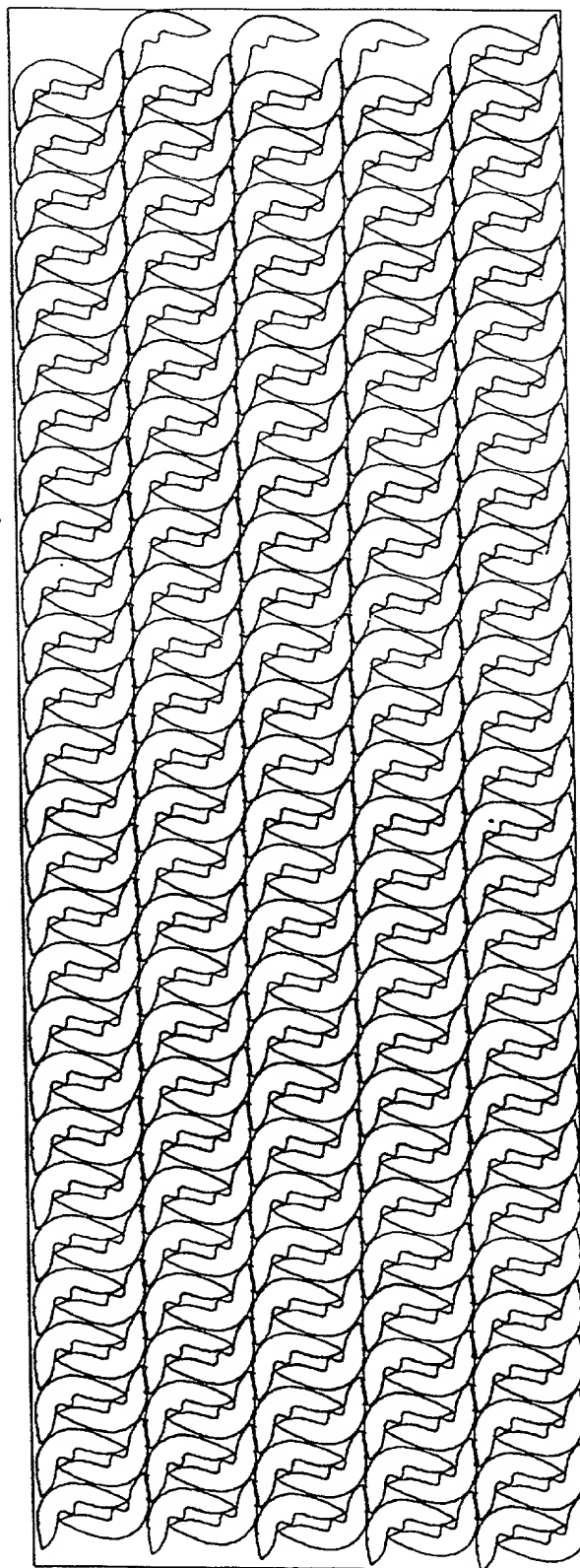


FIG. 10

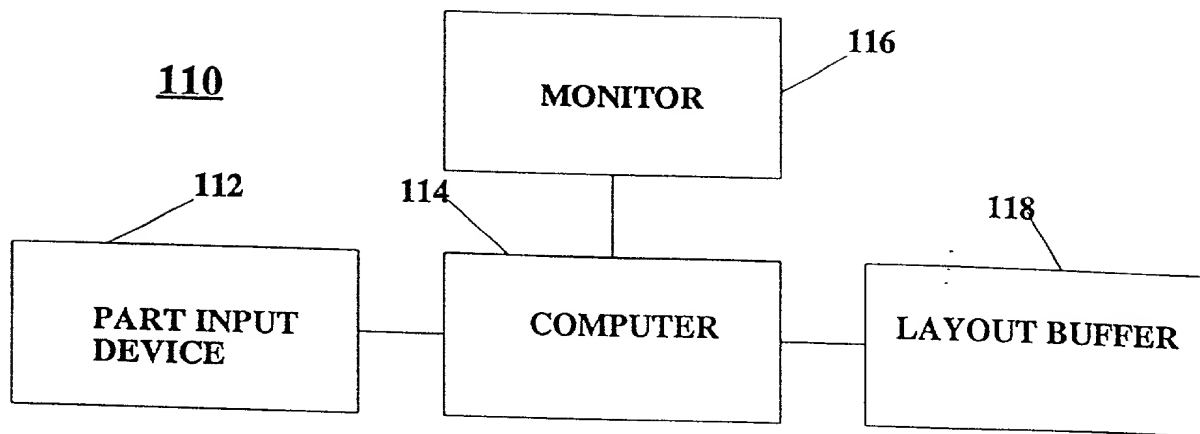


FIG. 11

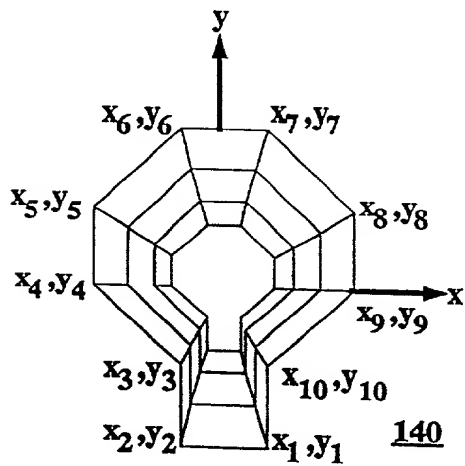


FIG. 12A

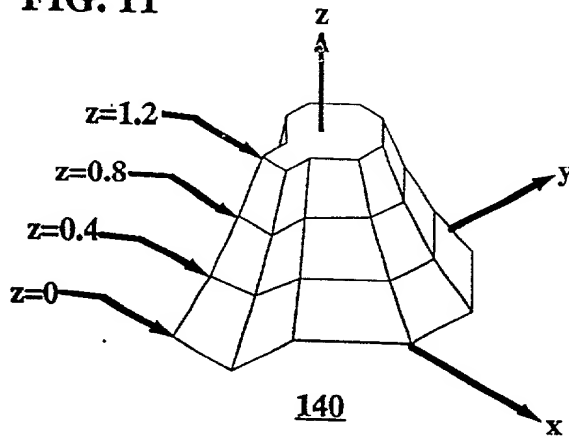


FIG. 12B

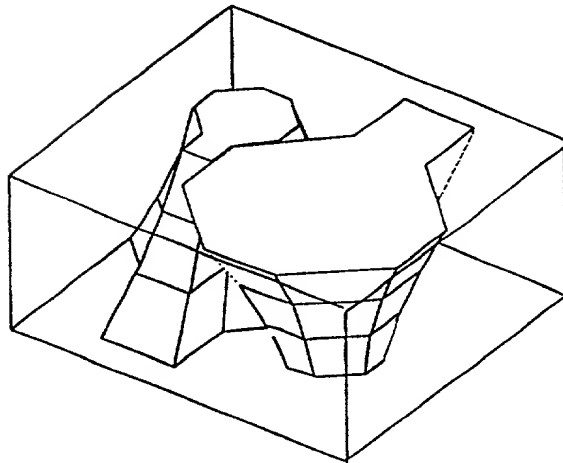


FIG. 13

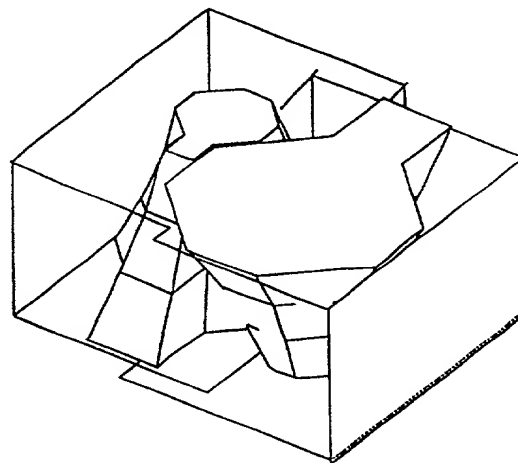


FIG. 14

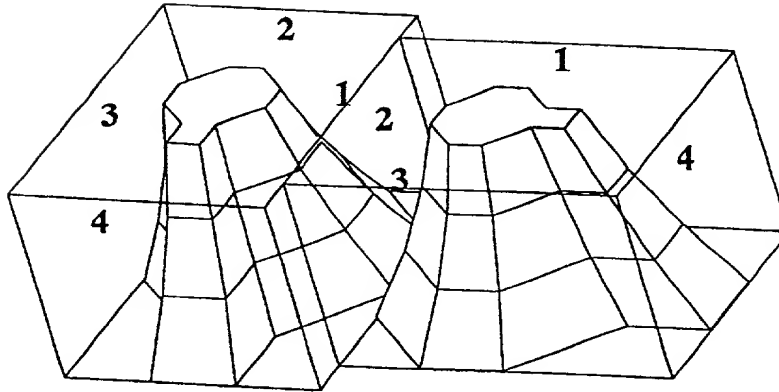


FIG. 15A

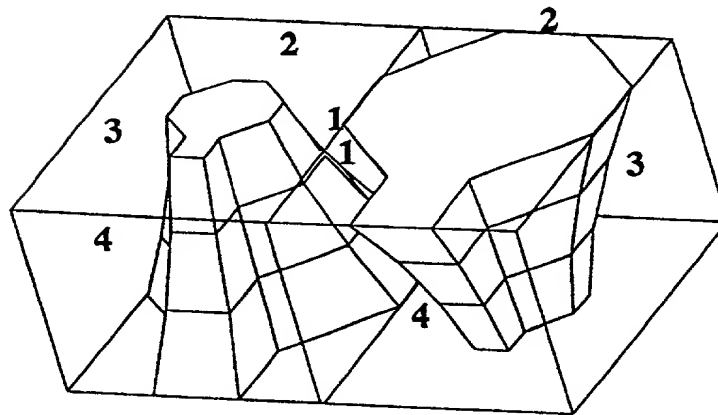


FIG. 15B

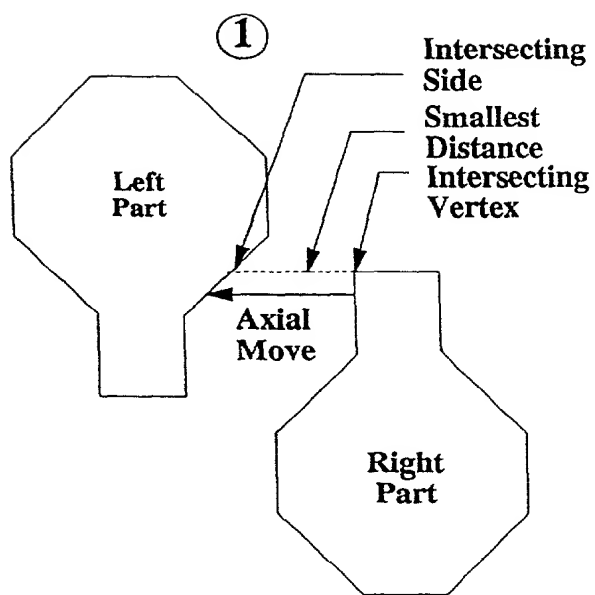


FIG. 16A

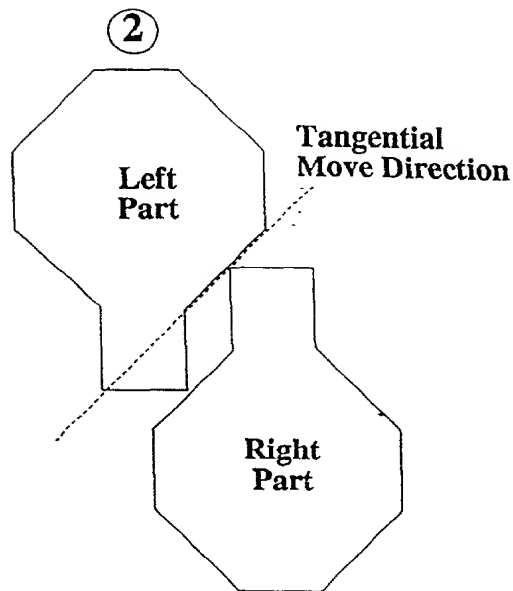


FIG. 16B

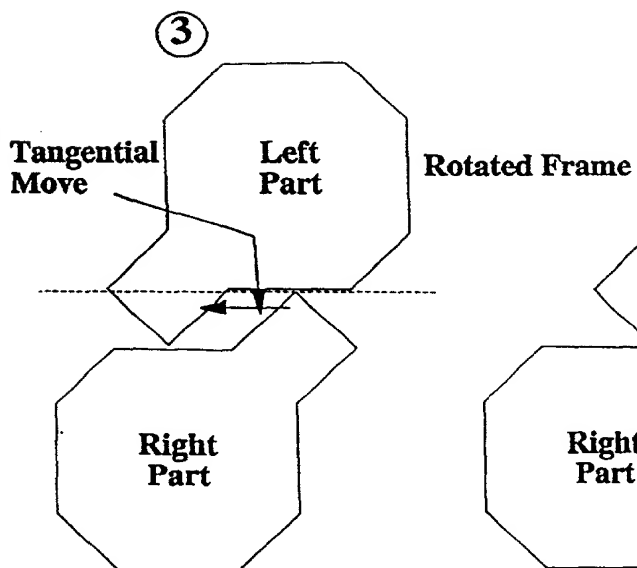


FIG. 16C

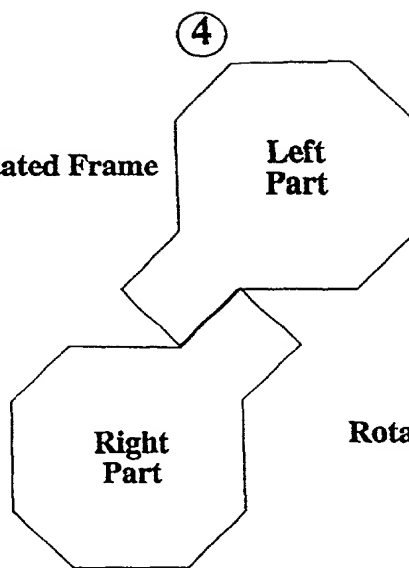


FIG. 16D

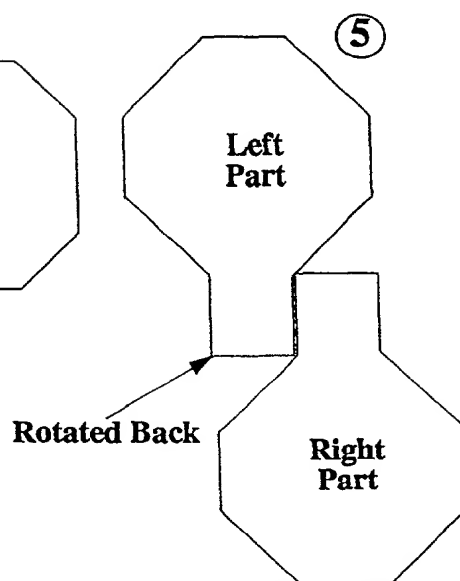


FIG. 16E

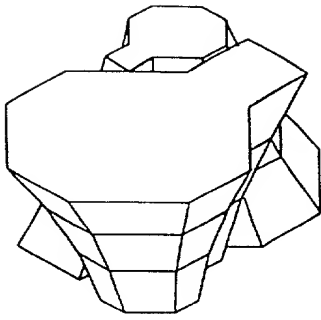


FIG. 17A

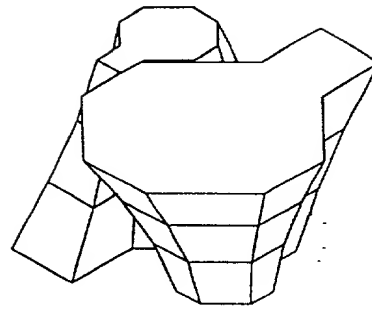


FIG. 17B

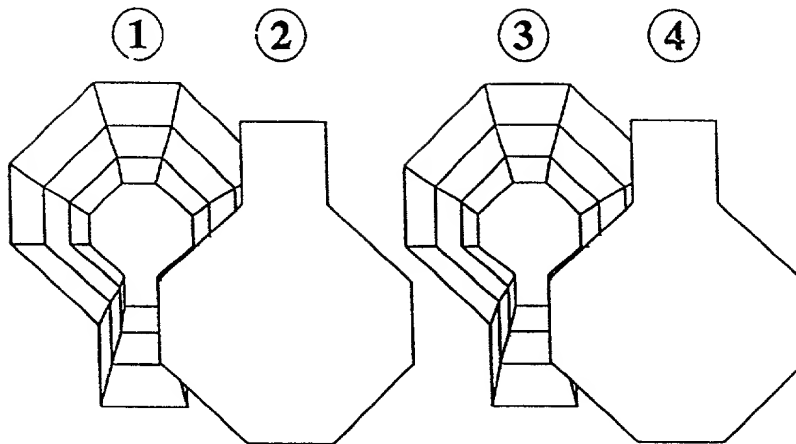


FIG. 18A

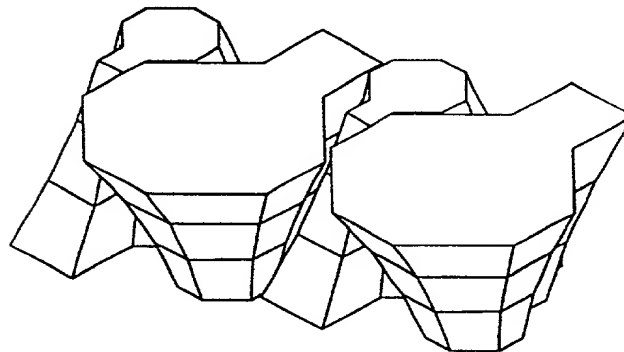
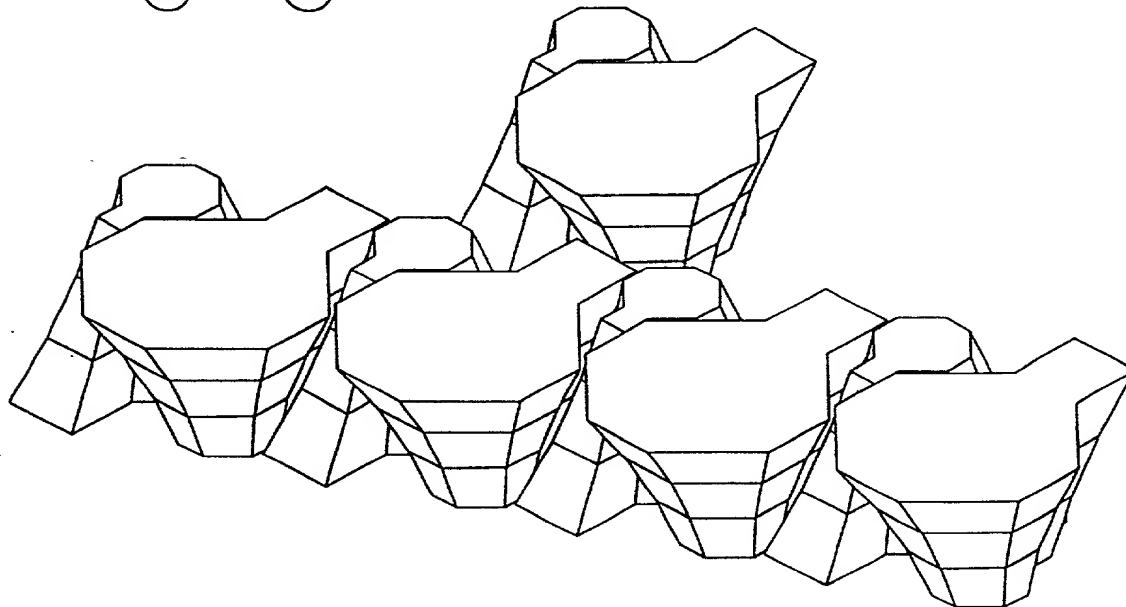
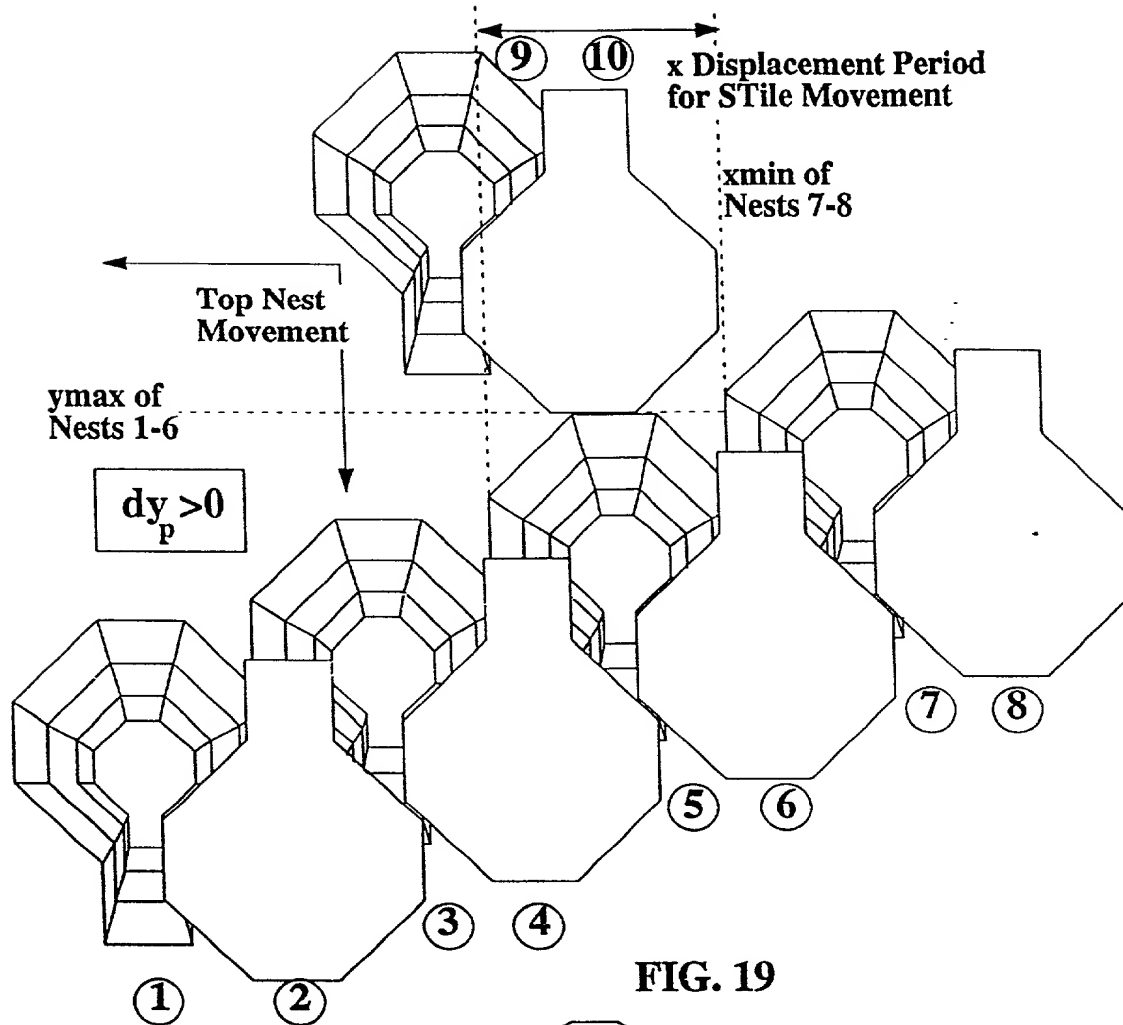


FIG. 18B



008727-50012500

Number of Nests/Ptiles Kept = 5  
Utilization Ratio = 0.346

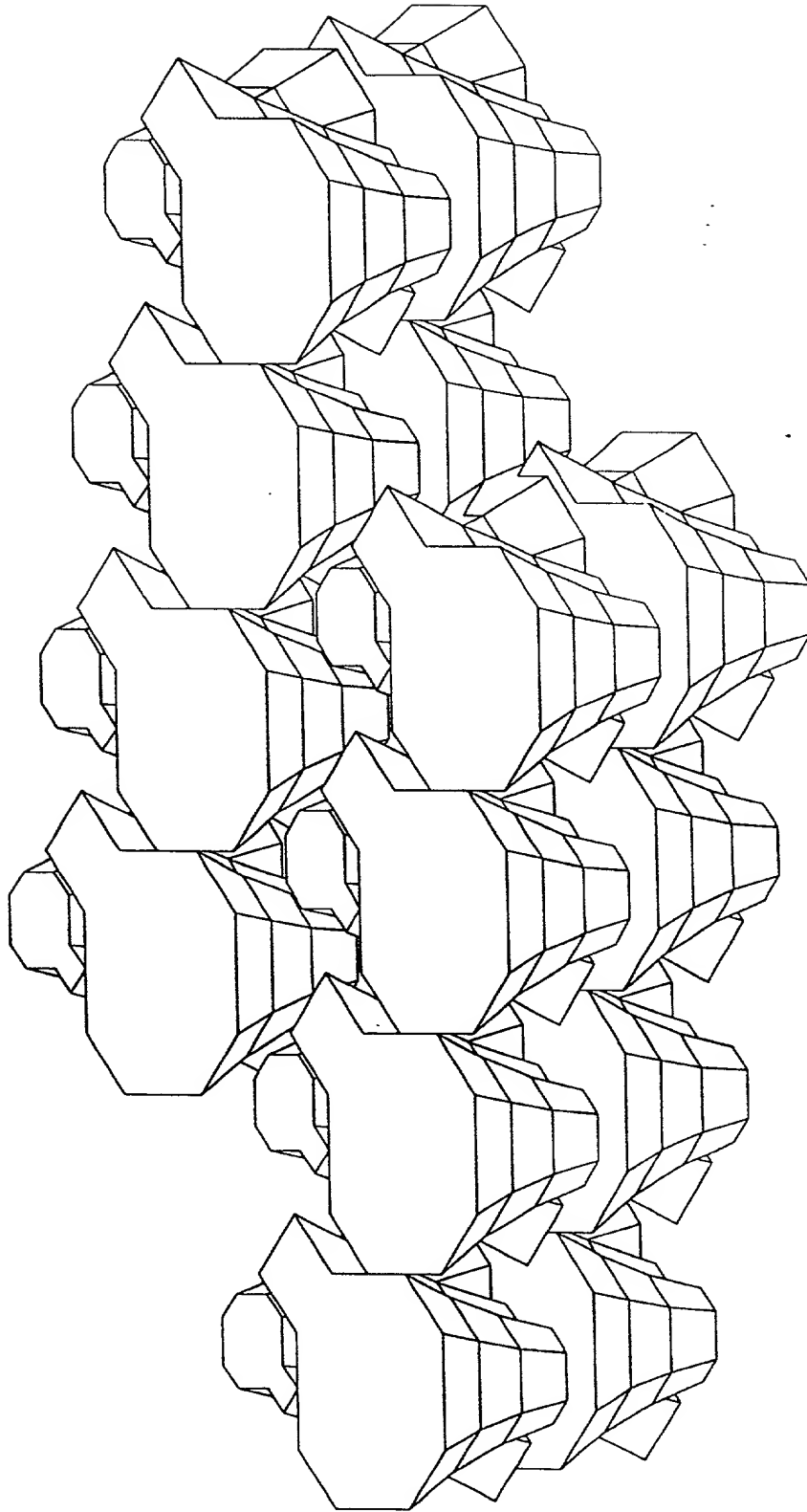


FIG. 21

002727 56078500

Number of Nests/Ptiles Kept = 10  
Utilization Ratio = 0.406

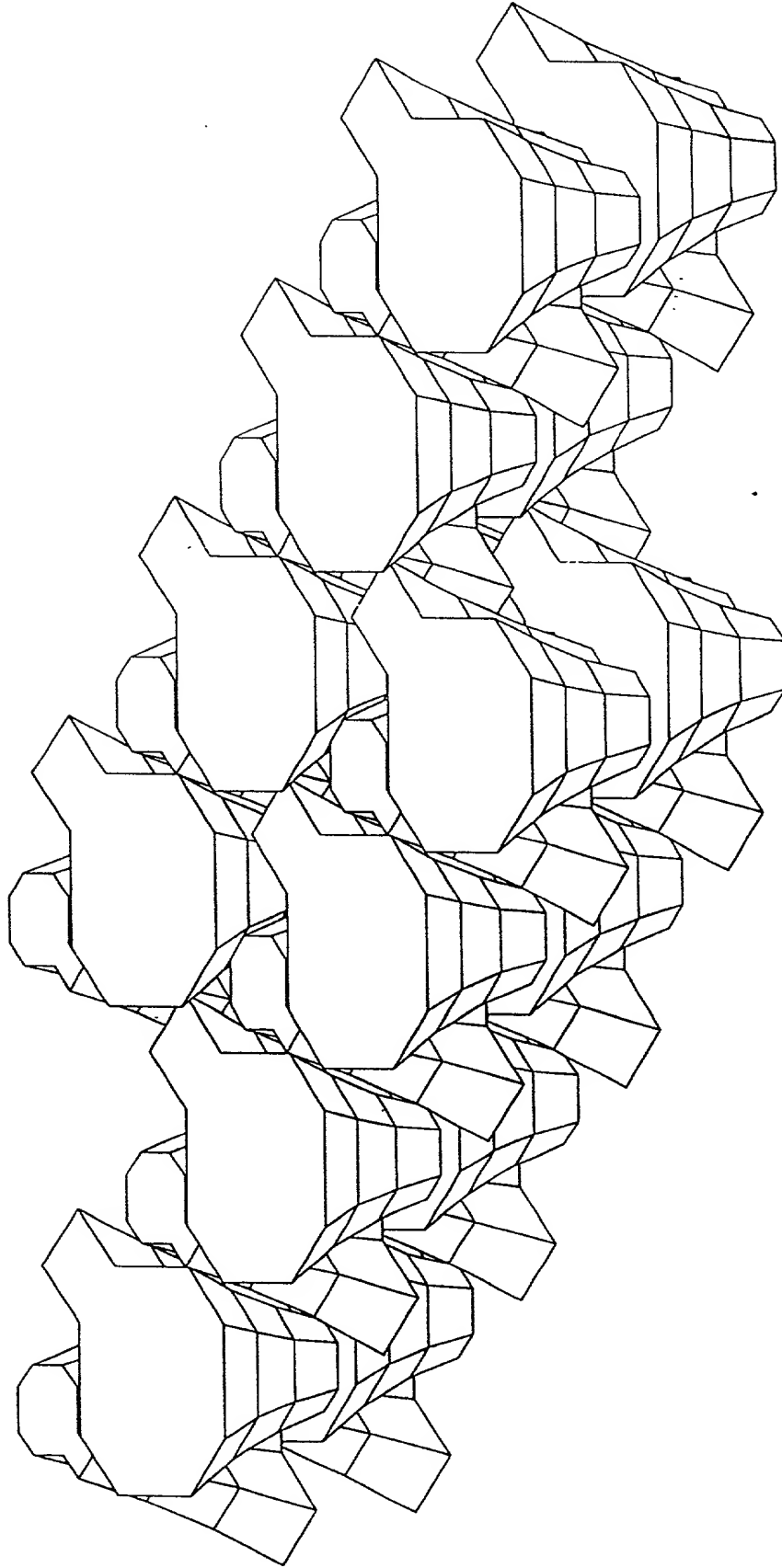


FIG. 22

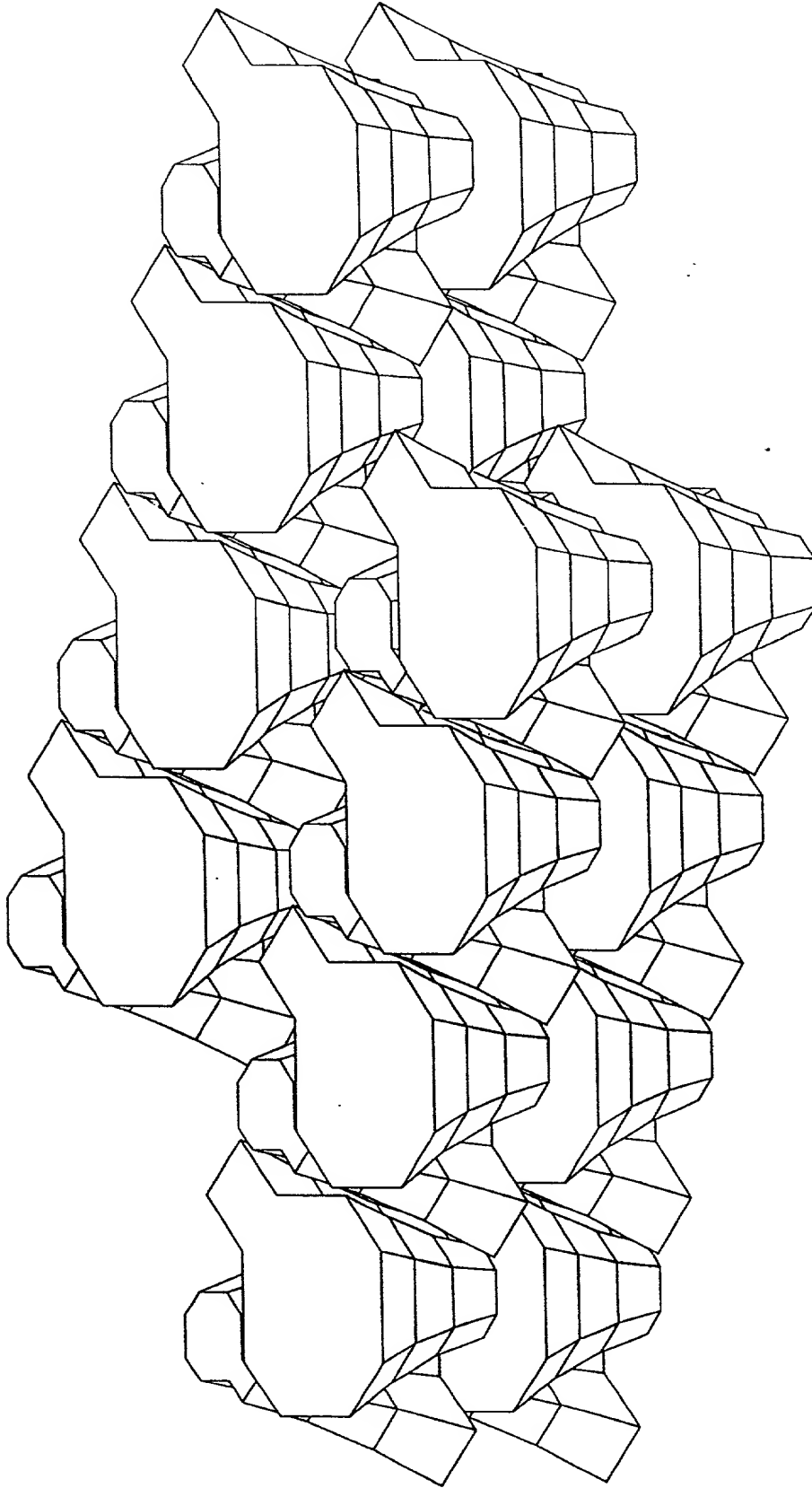
PCT/US 98/26384  
IPEA/US 12 JUL 1999

AMENDED SHEET



[illegible]

Number of Nests/Piles Kept = 20  
Utilization Ratio = 0.422



**FIG. 23**



Docket No.  
56345-013(GCLL-115)

# Declaration and Power of Attorney For Patent Application

## English Language Declaration

As a below named inventor, I hereby declare that:

My residence, post office address and citizenship are as stated below next to my name,

I believe I am the original, first and sole inventor (if only one name is listed below) or an original, first and joint inventor (if plural names are listed below) of the subject matter which is claimed and for which a patent is sought on the invention entitled  
**METHOD AND SYSTEM FOR NESTING OBJECTS**

the specification of which

(check one)

☐ is attached hereto.

☒ was filed on June 8, 2000 as United States Application No. or PCT International Application Number 09/581,095  
and was amended on \_\_\_\_\_  
(if applicable)

I hereby state that I have reviewed and understand the contents of the above identified specification, including the claims, as amended by any amendment referred to above.

I acknowledge the duty to disclose to the United States Patent and Trademark Office all information known to me to be material to patentability as defined in Title 37, Code of Federal Regulations, Section 1.56.

I hereby claim foreign priority benefits under Title 35, United States Code, Section 119(a)-(d) or Section 365(b) of any foreign application(s) for patent or inventor's certificate, or Section 365(a) of any PCT International application which designated at least one country other than the United States, listed below and have also identified below, by checking the box, any foreign application for patent or inventor's certificate or PCT International application having a filing date before that of the application on which priority is claimed.

Prior Foreign Application(s)

Priority Not Claimed

(Number)

(Country)

(Day/Month/Year Filed)

☐

(Number)

(Country)

(Day/Month/Year Filed)

☐

(Number)

(Country)

(Day/Month/Year Filed)

☐

I hereby claim the benefit under 35 U.S.C. Section 119(e) of any United States provisional application(s) listed below:

60/069,561	12/12/97
_____ (Application Serial No.)	_____ (Filing Date)
_____ (Application Serial No.)	_____ (Filing Date)
_____ (Application Serial No.)	_____ (Filing Date)

I hereby claim the benefit under 35 U. S. C. Section 120 of any United States application(s), or Section 365(c) of any PCT International application designating the United States, listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States or PCT International application in the manner provided by the first paragraph of 35 U.S.C. Section 112, I acknowledge the duty to disclose to the United States Patent and Trademark Office all information known to me to be material to patentability as defined in Title 37, C. F. R., Section 1.56 which became available between the filing date of the prior application and the national or PCT International filing date of this application:

_____ (Application Serial No.)	_____ (Filing Date)	_____ (Status) (patented, pending, abandoned)
_____ (Application Serial No.)	_____ (Filing Date)	_____ (Status) (patented, pending, abandoned)
_____ (Application Serial No.)	_____ (Filing Date)	_____ (Status) (patented, pending, abandoned)

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

10- POWER OF ATTORNEY: As a named inventor, I hereby appoint the following attorney(s) and/or agent(s) to prosecute this application and transact all business in the Patent and Trademark Office connected therewith. (list name and registration number)

Mark G. Lappin, P.C., Reg. No. 26,618

Toby H. Kusner, P.C., Reg. No. 26,618

Elizabeth A. Levy, Reg. No. 34,375

Ronald R. Demsher, Reg. No. 42,478

David M. Mello, Reg. No. 43,799

Jeffrey J. Miller, Reg. No. 39,773

Scott A. Ouellette, Reg. No. 38,573

Elizabeth E. Kim, Reg. No. 43,334

John Prince, Reg. No. 43,019

Debra A. Gaw, Reg. No. 38,463

Send Correspondence to: Mark G. Lappin, P.C.  
McDERMOTT, WILL & EMERY  
28 State Street  
Boston, MA 02109

Direct Telephone Calls to: (name and telephone number)  
Mark G. Lappin, P.C., telephone number 617-535-4043

Full name of sole or first inventor	
Isaac Sadovnik	
Signature of first inventor's signature	Date
<i>Isaac Sadovnik</i>	12/8/00
Residence	
117 Brackett Road, Newton, Massachusetts 02158	
Citizenship	
U.S.	
Post Office Address	
117 Brackett Road, Newton, Massachusetts 02158	

Full name of second inventor, if any	
Second inventor's signature	Date
Residence	
Citizenship	
Post Office Address	